Problem-solving skills in secondary maths education; what are the experiences, perceptions and reflections of practitioners.

By David A.W McLoughlin

Student ID: 12682389

University of Essex

A dissertation submitted in partial fulfilment of the

requirements for the degree of:

Masters in Education

“What is mathematics? It is only a systematic effort of solving puzzles posed by nature.”

Shakuntala Devi

Contents:

Abstract

The purpose of this study was to investigate the experiences and perceptions of British curriculum secondary maths teachers in teaching mathematical problem-solving skills, and to analyse these in light of the previous literature and research in the field. A thorough literature review was conducted and analysed to understand where the field of mathematical pedagogy currently stands with regard to effective problem-solving instruction. Then, to explore the lived experiences of practitioners, twelve semi-structured interviews were carried out with practicing experienced British curriculum secondary maths teachers. The interviews were transcribed and underwent thematic analysis to ascertain recurring themes present amongst the participants' subjective viewpoints. Exploration of the interview findings revealed that students' maths anxiety and comprehension are viewed by teachers as the two main barriers to progress in mathematical problem-solving. Furthermore, the pedagogical decisions made by teachers - when attempting to break down the barriers to mathematical problem-solving – differed from those commonly found in the literature. A comprehensive picture of the experiences and realities of maths teachers was uncovered. The data collected from this present study suggested that researchers should consider the concerns of practitioners more often when forming pedagogical suggestions, as currently, the priorities of teachers often differ greatly from the focus of prior research in the field. The current disconnect between research and practice opposes the notion that teaching should be an evidence-based profession. Therefore, this study argues that it is necessary for researchers and teachers to realign such that evidence leads to practical and applicable pedagogy that can be implemented effectively on the frontline of the profession.

Key Words:

Problem Solving, Maths Education, Teachers' Experiences, Teachers' Perceptions.

Acknowledgements

I would like to thank my supervisor Dr Angel Urbina-Garcia for his support and guidance, as well as the entire team at the Department of Education at the University of Essex who helped me throughout the entire Masters program. I would also like to thank the participants who volunteered their time to contribute to this research, on top of their hectic professional duties during a year of unprecedented upheaval in the education industry. Finally, I would like to thank the most supportive parents I could wish for, Gillian Denise McLoughlin and Kevin McLoughlin. Thank you for always being there for me.

Introduction

Problem-solving is an abstract concept that must be defined before continuing. Klerlein and Hervey and Klerein (2018:1) defined mathematical problem-solving as “any situation that must be resolved using mathematical tools but for which there is no immediately obvious strategy”. Barton (2018) identifies creativity and adept intuition as key tenets of problem solving. This definition suggests the need for intuition, resourcefulness, ingenuity, and adaptability, that sets mathematical problem-solving thought processes apart as an intriguing and rich topic for exploration (Giganti, 2007). Chamberlin (2008) considers that the many definitions of problem solving are convoluted, although he does state that problem solving tasks are necessarily non-routine. Considering all the possible definitions and concepts above, problem solving in this paper will refer to students attempting to understand a novel non-routine problem, develop a strategy and then successfully implement that strategy. This definition broadly captures the common themes discussed by the various abstract definitions above and remains true to the premises conveyed in the literature.

Problem-solving, especially in maths, is seen as a widely transferable skill which will become increasingly more valuable to a rapidly changing world and ever-changing jobs market in the 21st century (Szabo et al, 2020). UNICEF (2019) agree, stating that logical problem-solving skills are essential to the professional and personal lives of modern learners. A recent survey of employers’ perceptions reiterated this idea. Problem-solving was ranked highly amongst the most important skills necessary to compete in the current jobs market , as: “job seekers in the 21st century must be able to find logical solutions to the problems” (Suarta et. at, 2017). Economic analysis of the changing field of technology foresees a shift to many jobs becoming automated, especially repetitive manual skills

(PWC, 2018), emphasising the need for people to have robust and resilient skills which allow them to adapt to a changing industrial landscape and digitalisation. Fortunately, problem-solving is a significant element of mathematics education. In fact, problem-solving in mathematics has been linked to students developing a wide range of complex mathematics structures which are applicable to problems posed in real-life (Tarmizi and Bayat, 2012). It is therefore evident that problem-solving is a vital part of modern education and this research will seek to gain further insight into the reality of teaching problem-solving skills in the classroom.

With the above basis established, it became evident, upon literary review, that diverging methods to approaching mathematical pedagogy and problem-solving skills acquisition exist in the literature. The balance of the literature was heavily tilted toward experimental quantitative data, and this was often the justification behind many of the pedagogical strategies that emerged from the research. The lived experiences of teachers who constitute the frontline of the profession were largely unexplored (Catrambone and Eiriksdottir, 2011: Bokosmaty, Seeler and Kalyuga, 2015: Clarke et al, 2012: Hajar and In'am, 2017: Lessani at al, 2016). Problem-solving has been established as a salient skill for education and the empirical studies on the subject are wide ranging, however the phenomenological research from the perspective of the front-line practitioners is lacking. For this reason, in seeking to bridge this gap, the following research questions were formulated:

1. What are the perceptions of secondary British curriculum Maths teachers on their experiences of teaching problem-solving skills?

2. How do the perceptions of secondary British curriculum maths teachers compliment and conflict with the research base pertaining to mathematical problem-solving pedagogy?
3. How can teachers and researchers bridge the divide between theory and practice for implementing evidence-based pedagogy in mathematical problem-solving?

The focus of this research was chosen as the British curriculum due to the prominence of the British educational system internationally, as it is the most widely used curriculum in international schools worldwide (Clark, 2014). Moreover, the current UK and international GCSE syllabus rewards over 25% of marks to ‘non-routine problem-solving’ skills (Edexcel, 2014), hence the teaching of this curriculum lends itself to problem-solving pedagogy. Secondary maths teachers were specialists in the subject of maths and therefore had the greatest expertise in teaching the subject, hence this paper honed it’s focus on teachers whose students were this respective age range.

This present study will begin by reviewing the recent empirical literature concerning mathematical pedagogy, specifically the research that has sought to understand how problem-solving skills should be taught. Following this, the methodology by which further data was collected will be explained, along with the findings of this methodology. The findings will be discussed, with comparisons drawn to the literature review, to answer the research questions. Finally, a conclusion will offer insights into the questions and offer recommendations for practice and future research.

Literature Review

Mathematical thinking, learning and teaching are complex yet important issues, inspiring a large and growing research base over the last 50 years (Barton, 2018: Bjork and Bjork, 2009; Catrambone, 1998: Izzati and Mahmoud, 2018: Schoenfeld, 1992). Throughout this section, the contemporary influences on mathematical pedagogy will be explored, critiqued and analysed, thus laying the foundation for comparison with the views of maths teachers. With the aim of forming a comprehensive overview of the current scholarship associated with mathematical problem-solving.

Before exploring the pedagogical research on problem-solving instruction, a brief overview of the seminal literature on problem-solving in general will be summarised. Polya (1945) provided intricate and influential insight into how people solve mathematical problems, and his ideas are still being referenced in contemporary literature. Polya (1945) noted that mathematical induction is where generalisations are discovered from spotting patterns and can be used to solve escalating, more complicated scenarios. Reductio ad Absurdum, sometimes referred to as indirect proof, is the concept of showing something to be false by assuming it to be true to then arriving at an obvious illogical absurdity. Routine problems were defined as problems which largely resemble generic problems and require little original thought from a problem-solver. Polya then continued to give suggestions of where to begin approaching problems: draw a figure to visualise, isolate the unknown, consider a similar problem, rephrase the problem, look for useful information from the problem or prior knowledge and break it into simpler sub-problems. Contemporary research building on Polya's work echoes themes of conceptualisation, visualisation, application of prior knowledge and fluency with routine procedures (Mamona-Downs and Downs, 2005: Chapman, 2015: Maria and Carlos, 2007)

Now, it is apt to consider the prevalent educational philosophies that emerged in the 20th century. Behaviourism is an educational philosophy that proposes that all behaviours are learned through external environmental reinforcers; motivators and deterrents leading to ‘conditioning’ (McLeod, 2017). Behaviourists see students’ roles as passive, whereas constructivism suggests that people are capable of creating new knowledge through lived experience (Schunk, 2012), thereby encouraging experiences that lead to new ideas. Both perspectives have been influential, and the themes considered in these early theories resonate in the literature today.

Constructivism evolved into Bruner’s (1961) seminal theory of discovery based learning, which continues to influence current educational theorists. Discovery-based learning referred to students discovering and constructing their own knowledge, as opposed to being told directly by a teacher (Bruner, 1961). Two recent small-scale studies elaborated on Bruner’s work, investigating the effectiveness of discovery-based learning in maths classes. Hajar and In’am (2017) conducted lesson observations of six groups of high school students and considered the extent to which discovery-based learning was present, then compared the academic results of the six groups. They concluded that discover-based learning improved student results in geometry, however the reliability of this conclusion is limited by the sample size and should be verified with a larger and more rigorous experimental design. Another similar study compared two groups of students: a control group and a discovery-based learning group. The researchers observed that the discovery group was better motivated and achieved higher results (Kistian, Armanto and Sudrajat, 2017). The groups involved in this second study were grade 6 in an Indonesian school, and caution should be taken when making generalisations to older secondary students from vastly different socio-economic backgrounds. These results were reinforced with a decisively larger study which measured the extent to which several classes engaged in discovery-based learning and correlated that against their progress in problem-solving ability. The researchers found that a blended approach which integrated discovery-based learning into traditional teaching led to improved problem-solving skills (Herdiana, Sispiyati and Wahyudin, 2017). All three of the aforementioned studies remain limited regarding the precise nature of effective discovery-based learning, with no well-defined recommendations for effective implementation.

A 2010 study conducted in Taiwan investigated ‘inductive discovery learning’ in mathematical education, by observing inductive learning and measuring academic progress against a control group (Chang et al, 2010). The researchers defined the inductive discovery learning process; observing, identifying patterns, testing and generalising concepts, structured by the teacher but conducted by the students. This approach, they claimed, allows learning to be student centred and develops students’ independence, which in turn improved their adaptability and resourcefulness causing a noticeable improvement in their problem-solving performance (Chang et al, 2010). While there may be cultural differences that affect the mathematical ability of students in Taiwan versus the British curriculum (Eisenhart and Wei, 2011), the researchers’ pedagogical suggestions could still be shown to be applicable if validated by the opinions of teachers in this current research project.

Lessani et al. (2016) conducted a qualitative research study to compare applications of learning theories in maths education. They conducted lesson observations and semi-structured interviews with seven teachers. Their conclusions centred around the students' experiences as perceived from the teacher and the lesson observer. It was argued that problem-solving and discovery-based learning activities made students more active in lessons, made them more independent, lowered anxiety for exams and lead to improvements in their reasoning ability.

Discovery-based learning places the teacher as a facilitator to learning and allows students many degrees of freedom to explore mathematical concepts (Hajar and In'am, 2017). This may eventually lead to a holistic and complete understanding of maths, including an invaluable critical knowledge of which methods work, which methods are inefficient or which methods are wrong. However, this poses a problem as it is difficult to see how this free approach will converge upon the rigorous, refined and time-honoured mathematical methods that have been developed over the centuries. It is also unclear how the expertise of the teacher in these methods is communicated with the class. These well-defined methods are fast and efficient, and asking students to embark on self-discovery to uncover these methods is akin to reinventing the wheel.

The results of these small-scale studies are contradicted by the opinions expressed by some leading and well-respected educational psychologists (Clark, Kirschner and Sweller, 2006; Clark, Kirschner and Sweller, 2012). They argue that the tendency for educational professionals and some researchers to favour discovery-based learning is 'ideological bias' which has been refuted by 'half a century of empirical research'. This 'ideological bias' refers to teachers believing that discovery learning is inherently good. Elaborating, they explained that minimal guidance is especially ineffective for novice learners and potentially detrimental to learning as it allows students to form ‘misconceptions or incomplete or disorganised knowledge’ (Clark, Kirschner and Sweller, 2012). After extensive review of the literature in the field of cognitive psychology, Clarke et al (2012) endorsed the view that explicit teaching and guided instruction were far more effective than the minimal guidance from discovery-based learning (Clark, Kirschner and Sweller, 2012). Researchers have built on these ideas over the past decade, leading to conclusions which directly relate to problem-solving pedagogy.

Cognitive load theory, first proposed by Sweller (1988), has emerged to be a prominent pedagogical theory, with Wilam declaring it the “single most important thing for teachers to know” (Wilam, 2017). The theory considers the demands on working memory and the amount of information a student can process during learning. In brief, cognitive load theory suggests that a student whose cognitive load is overburdened will not form schema. However, a student whose cognitive load is efficiently focused on understanding a new concept, will be more successful in schema construction (Centre for Education Statistics and Evaluation, 2017). While cognitive load theory is praised for having a robust evidence base and the results consistently replicable, the theory has been inferred from randomised control trials without cognitive load being directly measured, which is seen as a shortcoming in the supportive evidence (Centre for Education Statistics and Evaluation, 2017).

The worked example effect is a direct corollary of cognitive load theory. It advocates for teachers to carefully and explicitly guide students through a new concept, predominantly through worked examples, especially during the initial introduction of new knowledge, as this approach minimises the cognitive load on students and allows them to process and store these ideas in long term memory more effectively (Bokosmaty, Sweller and Kalyuga, 2015). This has been shown to be effective in several studies, as described by Crissman (2006) in a meta-analysis, however this is outdated considering the rapid developments since, and a more current meta-analysis would be ideal especially considering the accelerating developments in the field. Bokosmaty, Sweller and Kalyuga (2015) conducted two experiments with a large sample of high-school students to compare the effectiveness of teaching with complete guidance, structured prompts or minimal guidance. The researchers convincingly demonstrated that worked examples are effective for students to develop complete understanding, as student academic results showed a noticeable improvement. The results showed that as the mathematical ability level of the students increases, the level of guidance should be reduced; novice learners need all the information clearly communicated to arrive at understanding, while higher ability students actually become distracted by processing the explanations they find obvious in a worked example. This appears to allow for a gradual increase in the independence of learners as they become more confident with the curriculum content. Additionally, the teacher has greater control over the progress of the lesson allowing for structured, organised and methodical knowledge acquisition.

Cognitive load theory and the worked example effect propose directly teaching prescribed methods, which is why they are so relevant to mathematical pedagogy. It is less clear how these ideas transfer adaptable problem-solving skills, as the prescribed methods are inflexible to novel scenarios. Cognitive load theorists suggest that this gap is not of concern, as mathematical procedures are acquired skills whereas problem-solving skills are innate to humans, not requiring input from a teacher (Geary, 2012; Tricot and Sweller, 2014). Problem-solving skills are seen as just applying logical thought processes and using mathematical methods, therefore problem-solving skills develop from practice once learners acquire a ‘toolbox’ of mathematical methods or procedures. This may make classroom teachers uncomfortable as they do not directly teach students to solve problems, merely providing them the tools to do so, therefore surrendering the control over important skills acquisition to the learner. This research paper will delve deeper into this concern.

Catrambone and Eiriksdottir (2011) suggest a solution which may allow teachers to be more involved in the development of problem-solving skills. The authors propose that the subgoal learning model can be effective at training student’s to logically problem solve through the delivery of explicit worked examples. They suggest that annotations added after a worked example is completed can signpost the ‘subgoal’ behind each step in the method (Catrambone and Eiriksdottir, 2011). As students become more familiar with the topic, the subgoals can become gradually more generic; this was shown to be effective in their literature review. Margulieux (2014) conducted a study which observed 120 novice undergraduate students attempt to program an unfamiliar program with unfamiliar software. The instructional learning was supported with annotated subgoals, as suggested by Catrambone and Eiriksdottir (2011). The results further supported the subgoal learning model by demonstrating greater conceptual understanding of the problem-solving strategy. However, the participants were more mature learners in a slightly different subject than the secondary students that this literature review focuses on, and results should be viewed with caution before generalising to other curriculum subject areas.

The subgoal learning model ties in with metacognition. Metacognition is defined by Izzati and Mahmudi (2018) as “the ability to monitor and control our own thoughts”. In their review of educational research on metacognition, they concluded that increased metacognitive ability positively correlates with increased problem-solving ability. Alzahrani (2017) conducted a study through lesson observations and found evidence to support metacognitive strategies in mathematics education and concluded that metacognitive strategies were best used to focus, monitor and regulate the thought processes of students. The study has some interesting suggestions about how to incorporate metacognition into maths education. They argued that metacognition should be explicitly taught so that students focus their metacognition at improving the efficiency of their thought process. Thus, the teacher should carefully plan and deliver metacognitive instruction.

Although Catrambone and Eiriksdottir (2011) and Alzahrani (2017) make no reference to each other's domains (the subgoal learning model and metacognition), there seems to be clear overlap in the intentions of their work. Are the two theories compatible and mutually supportive? The subgoal learning model seems to be a method of explicitly directing the thought processes in problem-solving, as metacognitive theory suggest would be effective. Furthermore, both tie to cognitive load theory by explicitly teaching concepts and expanding on it by offering ways of improving student's problem-solving ability. There is a lack of research or commentary in the literature about the link between metacognition and discovery-based learning, but if a student was able to discover a solution to a problem themselves and then reflect on their thought-process, this would seem like a powerful learning experience. It is surprising that this has not been explored, and it is recommended that future research aims to tie metacognitive skills and discovery-based learning.

Whether explicit instruction or discovery-based learning is used to develop student knowledge in a new topic, the students will presumably eventually become more independent as they attempt and practice problem-solving themselves. A suggested technique to increase their awareness of the processes they are utilising is the self-explanation effect. Supporters of the self-explanation effect suggest that prompting students to pause and consider the steps in a problem, explaining to themselves, often mentally, helps them process the logical reasoning behind the maths they have used (Bisra et al., 2018). A meta-analysis of research concerning the self-explanation effect has shown that the technique can be effective in maths education, although Durkin (2011) states that the effect was often overstated or exaggerated. Another study by McEldoon et. al (2013) looked at the self-explanation effect from a time-efficiency perspective. The experimental design was well suited to test the self-explanation effect against other options for productive lesson time. The researchers found that the self-explanation effect has some modest but unique benefits, compared to using the same learning time for additional practice. The authors did note that the key to the self-explanation effect is not clear articulation of the exact logical process behind their action, which would be far beyond the ability of most secondary students, but the attempt to actively process what they are doing, not passively following algorithmic steps (McEldoon et. al, 2013).

The literature described above agrees that the self-explanation effect connects independent student work with metacognition. If this can be implemented effectively, perhaps this is a promising avenue to explore to allow students to become more independent while retaining the beneficial learning outcomes of guided instruction. The literature fails to offer clear suggestions for implementation of the self-explanation effect, and this highlights another disconnect between theory and practice.

The literature focuses on self-explanations but fails to explore the role of peer-explanations. Two students who have different levels of understanding of a concept may have a mutually beneficial discussion, with the student with a stronger understanding benefitting from the principles of the self-explanation, whilst also giving an individualised and responsive explanation to a struggling student.

Rote learning, sometimes called automation in mathematics, is when a student learns mathematical facts well enough to be able to automatically recite the answer without effort. Mhlolo (2015) acknowledges that rote learning is often 'disparaged' in the teaching community but asserts that it is sometimes a necessary foundational step to allow for deeper learning. Narayan (2009) praised rote learning as effective preparation for higher level concepts. Rote learning is widely practiced in the curriculums of India and China, however, attempts to see if this approach can be successful in the west were unsuccessful (Brown and Roy, 2013). Presumably the ability to know an answer to a basic mathematical operation without conscious effort will lower the overall cognitive demand on a student during a more complex mathematical task, and hence be in line with the suggestions proposed by cognitive load theory. Although the process of rote learning may undermine the philosophy of maths education - that maths should be conceptually well understood by learners - perhaps the time spent rote learning number facts will allow complex multistep procedures to become more accessible. As cognitive load theory implies, the less students have to think about each detail, the more they can think about the broader task.

Finally, the themes discussed above centre around a pedagogical perspective and consider how students develop problem-solving skills. As rich and as interesting as this is, it detaches students and education from the role of emotions in maths education. After all, teenagers experience a range of complicated and intense emotions (Bailen et. al, 2018). In the broader literature, there is another barrier to mathematical achievement that appears:

maths anxiety. Maths anxiety refers to a fear or apprehension of situations which involve maths (Beilock and Willingham, 2014). A literature review found that maths anxiety negatively affects some students in most classrooms, and this has a negative impact on their ability (Anthony and Whyte, 2012). The emotional support a teacher can offer for students struggling with maths anxiety is suggested to be an effective intervention to improve performance in maths (Beilock and Willingham, 2014). The authors suggested several ideas for reducing maths anxiety in students namely: ensuring fundamental skills, adapting assessments to reduce pressure on students, actualising emotions and considered phrasing of reassurance (Beilock and Willingham, 2014).

Beilcock and Willingham's (2014) humanistic view of learning in maths is a refreshing change from the cognitive psychology found elsewhere in the literature (Catrambone and Eiriksdottir, 2011; Bokosmaty, Sweller and Kalyuga, 2015; Alzahrani, 2017). This research paper will examine this humanistic approach to see if it is more aligned with the views of the maths teachers; will they favour a humanistic approach or ground themselves in strict scientific pedagogy?

The themes discussed in this literature review show that there are several proposed strategies for teaching and improving problem-solving skills in secondary maths students. These strategies have varying levels of support, with cognitive load theory receiving the clearest praise from the most respected experts in the field of educational psychology. Explicit teaching and discovery-based learning continue to be the two prominent teaching strategies, after decades of debate in the research as well as in practice. Metacognition is seen as a powerful tool for students to become aware of their thought-processes, as they are supposed to utilise this skill to improve their problem-solving prowess. Rote learning has limited support in the research; however, it potentially complements other well supported theories, including cognitive load theory.

It was particularly thought-provoking when Clark, Kirschner and Sweller (2006) critiqued discovery-based learning as ideological, despite other researchers finding supportive evidence of discovery-based learning methods. Their position should be carefully considered when interpreting the opinions and experiences of maths teachers in this study, as opinions are biased and susceptible to ideology.

The majority of research papers discussed above have taken an experimental approach with quantitative data, with the noticeable exception of Lessani et al (2016) who did investigate the opinions of maths teachers when arriving at their conclusion. There is evidently a disconnect between experimental evidence which is often results focused and the actual experiences of maths teachers who are on the frontline of the profession. Furthermore, few research papers gave clear, concise and directly applicable pedagogical strategies for teachers to implement but described the general theories to construct strategies from. Only one research paper described above considered exploring how efficient a teaching strategy was compared to other uses of class time (McEldoon et. al, 2013). Teachers will have valuable experience of judging how to use class time practically and efficiently to achieve the learning objectives. Hence, the need to explore the opinions and experiences of maths teachers who bare the pressure of teaching every day.

Methodology

The scientific method rests on the principle that results must be replicable and repeatedly verifiable, as this is the foundation for our confidence in new knowledge (Joffe, 2007). Throughout this section, the research methodology will be described and discussed so that the rigor of the results can be trusted. As this is a qualitative research study based on subjective opinion, it is unlikely that a similar study could replicate similar results however confidence in the credibility, veracity and dependability of the results is nevertheless vital (Guba and Lincoln, 1985).

Research Philosophy

This research project focuses on the experiences, perceptions and opinions of maths teachers. Therefore, a research methodology was employed which was most effective at gathering detailed, accurate, reliable, relevant, useful and interpretable information directly from teachers. The literature that has been prevalent in the field of mathematical pedagogy has arisen from a positivist worldview, supported by empirical scientific data. These results have been rigorous and sometimes exemplary in their scientific integrity; however, they are often constrained by the positivist worldview that fails to address the unique subjective views of individuals (Opie and Brown, 2019). As stated in the research aims, this research project will aspire to make a small contribution to the field by unifying the results of differing research paradigms in the field of mathematical problem-solving pedagogy, or perhaps uncovering deviation between researchers and practitioners. This will be done by contributing empirical evidence of teachers’ perceptions and experiences of teaching problem solving skills.

Quantitative data is insufficient to explore the full complexity and intricacy of a person's opinions (Cohen et al., 2018; Given, 2008). Opinions are subjective judgements and best expressed through dialogue (Wellington, 2015). Several research methods were considered, including mass surveying and structured interviews, however these were deemed inadequate at delivering the detail needed to uncover and probe the complex opinions of participants. Semi-structured interviews were chosen as they offer opportunities to rigorously understand the ideas expressed by participants, while also being flexible and responsive to the ideas that participants expressed (Cohen et al, 2018). Semi-structured interviews allow participants the freedom to express their views openly while remaining focused on the aims of the research project (Pharm, 2014). Semi-structured interviews remain practical for the sample size of 12 participants, given the time constraints on a single researcher (Opie and Brown, 2019). Since the subject matter is wide ranging and some perspectives might not have been considered during the interview preparation, the flexibility afforded by semi-structured interviews allowed for discussion of novel and unpredictable concepts introduced by participants (Cohen et al, 2018). Semi-structured interviews allowed some degree of flexibility while remaining sufficiently structured to ensure enough of a framework for consistent and coherent themes to be uncovered in the analysis, while remaining relevant to the research objectives. Another crucial benefit of taking this approach is that it contrasts with the overwhelming majority of quantitative and impersonal research which has been previously conducted in the field, as observed in the literature review. This phenomenological approach could substantiate the empirical research in the field, or could highlight shortcomings in previous literature, therefore making a positive and constructive contribution to the field.

Sampling Methodology and Participant Recruitment

The inclusion criteria would be that the volunteers must have completed initial teacher training in an accredited institution (Department of Education, 2020). This was to ensure a level of expertise which would allow for professional and well-considered opinions, in turn ensuring that the data collected was representative of the experiences of recognised maths teachers. Additionally, participants must have had at least 3 years of experience teaching maths in British curriculum schools because this would raise the quality of the discussion, as participants could draw upon a richer experience in the profession. There was no other inclusion or exclusion criteria. Constraining the sample based on other factors may alter the demographics and unfairly prejudice valid professional opinions. This sampling method was sufficient to address the research questions. Indeed, care was taken to diversify the sample to balance for gender, age, cultural background, religion or other factors that could have influenced their outlook on maths education. Networking online, as opposed to interviewing professional colleagues or other convenience sampling methods, offered a more randomised sample and avoided imparting bias (Adams, 2015). While it was not feasible to stratify the sample to be truly representative off all British curriculum maths teachers, the sample was not homogenous and represented a diverse cross section of teachers. A sample size of twelve participants was chosen to give sufficient breadth of opinions while remaining feasible in the timeframe (Fusch and Ness, 2015).

The reason for focussing on the British curriculum is because it is a reputable international curriculum which was familiar to the researcher/interviewer, while also having problem-solving at the core of the curriculum (Edexcel, 2014). Furthermore, the research discussed in the literature review rarely referred to specific curriculums, and no instance was found that directly referred to the British curriculum. Nevertheless, worldwide, the British curriculum is taught in 43% of international schools making it the most popular international curriculum outside the host country (Clark, 2014), a testimony to its international reputability and credibility.

A recruitment process was conducted through networking online. Participants were told clearly and upfront the time commitment and expectations. If they chose to respond to the public posts, they were sent a participant information sheet, personal consent form (see appendix) and asked for a convenient time to conduct the interview online (BERA, 2018). There was complete transparency throughout, all the correspondence clearly communicated the strict ethical criteria which the project would and did adhere to, in line with the policies of the University of Essex (2018) and BERA (2018).

Interviews were scheduled at the convenience of the participants. Convenience was key to ensure that they were calm and relaxed during the discussion, which should lead to more detailed, clear and well-contemplated answers. Additionally, all participants were busy professionals volunteering their time during a year of unprecedented challenges as teachers were forced to adjust to the Covid-19 pandemic, and the researcher's view was that the experience of participating in this project should be a pleasant conversation not burdening them with further workload or pressure. This research project aims to positively contribute to the education industry, and for that reason the methodology was designed to have minimal disruption to participants' professional lives.

Interview Design and Execution

The semi-structured interviews were designed to evoke genuine and accurate reflections on the participants' professional practice, and care was taken to avoid leading or suggestive questioning so as to avoid imparting bias (Adams, 2015). The semi-structured nature of the interviews allowed a coherent framework for the questions, while remaining adaptive to the ideas expressed by the participants. By maintaining a consistent framework for the interviews, the researcher was able to remain transparent with the questioning and avoid bias (Opie and Brown, 2019). The questions and follow-up questions are available in the appendix, along with the rationale for the choice of questions.

The interviews were conducted online through video conferencing, which was imperfect but necessary during the Covid-19 pandemic. Regular face-to-face interviews benefit from non-verbal communication, resulting in more information being communicated to the interviewer (Ryan et. al, 2009) therefore increasing the amount of data gathered, however this was not possible at a time when social distancing was required. Therefore, Zoom was chosen to host the interviews as it is reliable and offers high quality video and audio, which should mitigate some of the shortcomings of the online interviews, particularly the non-verbal communication being more visible. During the interviews, field notes were taken as field notes helped to capture the tone of the comments in the moment which may be overlooked later. Interviews were recorded and later transcribed, to undergo thematic analysis. Interviews were between 30 and 40 minutes long, and this was strictly regulated to avoid inconveniencing participants. Longer interviews may have afforded more detailed discussion, however this would have been impractical for one interview given the short time constraints on the project.

Data Analysis

Thematic analysis is a technique to analyse qualitative data, by identifying patterns or themes in the data (Maguire and Delahunt, 2017). The transcripts and field notes were examined to see if recurring themes were present, or if there were differing ideas between participants (Clarke et al., 2015). Once common themes had been identified, they were tabulated and categorised. Quotes were isolated that evidenced recurring key themes around mathematical problems-solving teaching and this is discussed in the findings section. The transcripts yielded many themes, however salient themes were identified that were most prevalent and relevant to the research aims.

Further Limitations

The small-scale nature of this study causes several limitations. As the study focusses on the perceptions of teachers, it does not evaluate their actual quality of teaching or the merit of their pedagogical strategies. Conclusions can still be drawn from the practicalities and realities of implementing teaching strategies concerning problem-solving in maths, but these conclusions will not inform whether the perceived effectiveness of teaching strategies translates to positive learning outcomes. This leaves room for future research. The opinions of teacher's could be used to inform pedagogical practice, but would this lead to improved progress in problem-solving skills? Advocates may argue that teachers are the frontline of the teaching professional and best placed to judge learning outcomes, whereas sceptics may warn against pandering to the whims of teachers and focus entirely on the data from quantifiable educational research.

Ethical Considerations

Throughout the research, ethical guidance from the University of Essex (2018) and BERA (2018) was strictly adhered to. This research aims to positively contribute to the education industry, and so ethical considerations were considered so that no inadvertent harm was caused to educational professionals or any other person. Participation in the project was completely voluntary, and the participants had the right to withdraw at any stage (BERA, 2018; University of Essex, 2018). The withdrawal process was simple and clear to all participants from the start (BERA, 2018; University of Essex, 2018). Informed consent was clearly communicated with consent letters (see appendix), which were signed and returned to the researcher prior to any interviews. The timescale, expectations and research process were also clearly communicated (see appendix). Data gathered on individuals was anonymised and no identifiable information was gathered as it was unrelated to the research aims (BERA, 2018; University of Essex, 2018). The data gathered during the interview was stored securely on a password protected device (BERA, 2018). A risk assessment was conducted and due to the nature of the short and professional interviews, it was deemed that the risk of physical or emotional harm was very low (BERA, 2018; University of Essex, 2018).

Findings

In this section, the observations from the interviews and transcripts will be reported, to be later critiqued and discussed in the discussion section. After the twelve interviews were conducted and transcribed, the interviews were reviewed, and connections made between them. Five themes emerged from the cohort of participants: maths anxiety, comprehension as a barrier to mathematical problem-solving, the role of the teacher, contextualising maths and teaching strategies and approaches. These themes were identified by connections between recurring ideas from the participants during the thematic analysis.

The thematic analysis was conducted methodically. When several teachers expressed ideas that could be linked together through some umbrella concept, specific quotes were taken and tabulated under that umbrella concept (see Appendix 3). The umbrella concepts (themes) emerged from careful analysis of the transcripts and interviews, as it became apparent which themes were repeatedly discussed in response to the interview questions. The quotes and opinions expressed may agree or disagree, showing a range of views on a specific subject, nonetheless exposing common concerns, ideas, thought-patterns, experiences, perceptions or areas-of-interest that could be of significance to the research aims.

Theme 1: Maths Anxiety

Nine out of the twelve (75%) participants directly mentioned maths anxiety, or described an emotional response hindering ability akin to maths anxiety. Two other participants (16.7%) briefly mentioned that students dislike maths. The emphasis that multiple participants placed on this theme and the regular reoccurrence highlighted the prominence of maths anxiety as a concern for maths teachers, and for that reason it is the foremost theme to be reported.

Maths anxiety was seen as a psychological issue, an emotional block which hindered students from progressing with problem-solving tasks even if the task was well within their ability. One participant stated “It [maths anxiety] is a massive problem, it really stifles the kids.” Another participant noted, “Once they’ve done it, it doesn’t seem so bad.” This implied that the students were mathematically capable of accessing and completing the problems, however it was an emotional barrier holding them back, not an intellectual barrier. Indeed, four other participants described similar opinions that the emotional response was where the problem lay. Two participants saw this as an opportunity to teach resilience or perseverance, with one teacher describing these two traits as “life skills”. Nine out of twelve (75%) of the teachers showed empathy with their students’ regarding maths anxiety.

Teachers explained how they tackle maths anxiety. In some cases, they lowered other barriers to maths which in turn made the task more accessible and less overwhelming (see theme 2). For example, one teacher specifically mentioned ‘scaffolding’ to make tasks more accessible to individual students as a direct response to their tendance of maths anxiety. Two teachers (16.7%) detailed the focus they give to the nurturing and pastoral side to teaching. They saw themselves as mentors who encouraged young people who lacked confidence. This was demonstrated by Participant G7; “I think the pastoral side of teaching can really help.” When asked for the impact on progress, they confidently replied: “yes, I have seen it improve progress.”

Theme 2: Comprehension as a Barrier to Problem-Solving

Students having difficulty understanding or processing the information given in the question was a common theme reported by the teachers. Ten out of twelve of the teachers (83.3%) mentioned they encounter pupils who find understanding the wording of questions difficult, while six out of the ten who mentioned this (50% of total cohort) described this in detail as a significant barrier to progress in maths. Three of these participants highlighted how this is particularly prominent with EAL and SEN (Special Educational Needs) students, as evidenced by Participant G7 who stated "I teach a lot of EAL learners, they don't like wordy questions. I have to spend a lot of time breaking down the sentences with them."

Several strategies were suggested to make the questions more accessible to students. Three teachers (25%) suggested highlighting or underlining the 'command words' in the question. Command words referred to the words that were instructional and could guide the students. Two teachers (16.7%) recalled how they had attended professional development sessions to work on exam technique and question comprehension. The sessions had suggested using acronyms to help students remember how to answer questions; one teacher described this technique with unconvincing conviction in the strategy while the other said that it overcomplicated the process of reading a sentence. There seemed to be low support and scepticism for acronyms as a strategy to guide comprehension.

One teacher (8.3%) said "I reveal the first sentence of the question, we discuss together. Once they are happy, we continue with the second sentence." This teacher described how they design their PowerPoint presentations to reveal one sentence at a time to avoid overwhelming blocks of text. They were visibly enthusiastic about this technique and were able to recall lessons where it had been effective at allowing students to understand convoluted questions.

The discussion around comprehension often overlapped with maths anxiety. As Participant K11 stated: “[when] pupils see questions in exams with long paragraphs, they just turn the page.” These two themes constitute the two most prominent barriers to students’ mathematical problem-solving progress that were described in the interviews. Often, these two ideas were the first thoughts that teachers expressed when asked about the difficulties to teaching problem-solving.

Theme 3: The Role of the Teacher

Throughout the interviews, there were many instances of teachers referring to themselves as a ‘lecturer’, ‘facilitator’, ‘mentor’ or ‘advisor’. One teacher did mention a specific scenario involving project work, in which they took the role of ‘head of the advisory board’. There was a large degree of variation in the differing roles of the teacher in the classroom, often this decision was based upon the learning objectives for that lesson.

Three teachers (25%) spoke of how they favour reserving discovery-based learning for higher ability classes, as lower ability classes need more structured and guided activities. As stated by Participant J10: “Lower classes struggle with independent work, I would only use it [discovery-based learning] for top sets”. Participant C3 (8.3%) praised discovery-based learning as akin to the scientific method, similar to how science experiments are used in science classrooms. They stated that their primary objective was for students to become adept at spotting patterns, and in their opinion, this was linked to problem-solving skills; however, the exact nature of this link was not clearly explained.

Five teachers (41.7%) mentioned either explicit teaching or lecturing as their preferred teaching style. “Worked examples are so powerful” and “Lecturing gives me control” were examples of teachers praising this teacher-centred approach. However, Participant G7 did acknowledge the limitations to this approach: “It can be a double-edged sword. Sometimes I have to model the expert way of doing things, but sometimes the students get scared to ask questions if I talk like that. They feel like it’s not a safe space.” The ‘safe space’ comment was echoed by other individuals who also considered how their role in the classroom could impact on student’s maths anxiety.

There was no overall majority or consensus on how teacher focussed, or student focussed lessons should be. There were factors that influenced the decision, such as: ability level, topic, learning objective, engagement, or the students’ personalities.

Theme 4: Contextualising Maths

Seven out of twelve (58.3%) of the interviewees specifically mentioned examples of where maths could be applied in students’ daily lives or in an applied activity. Three of these teachers alluded to the abstract nature of maths being a demotivator for students, as students do not understand the reason that they are being forced to learn this material. Participant C3 said “They always ask ‘where will we ever use this?’, they don’t understand why the maths is useful.”

There was some debate over the use of calculators in maths class. Three teachers (25%) argued for retaining mental arithmetic or traditional pen and paper calculations in lessons and homework throughout school. As Participant J12 said “Students just don’t have the foundation skills necessary to access the curriculum.” Similarly, Participant B2 said “Fluency with basic maths skills definitely makes problem-solving easier.” Participant A1 said “I encourage pupils to estimate an answer before trying it.” Estimation is a skill involving mental arithmetic and therefore it is reasonable to interpret this as encouraging non-calculator skills. Two teachers (16.7%) countered that the current curriculum is assessed entirely through exams which allow calculators, and therefore the teaching should match the curriculum and final assessment objectives. Only one teacher (8.3%) saw calculators as the modern way of approaching maths and by embracing the use of calculators in schools, pupils will become more prepared for the modern world. Three teachers (25%) mentioned that using a calculator is a skill that must be taught.

Two teachers (16.7%) offered novel ideas to counter what they called the ‘abstract nature’ of the maths curriculum. Participant H8 spoke of a cross curricular project of interior design which allowed students to apply maths, business and art skills. This participant described this as ‘real world problem-solving’. Participant K11 used creative activities to connect maths to the real world. They gave one example where the students were baking a cake in maths class to help visualise and demystify area and circumference. They said that this approach was engaging for students who were disenfranchised with maths or school.

Participant F6 said “They often know individual steps, but don’t know how they all fit together.” This revealed an insight into how maths skills can be learned without an awareness for what these skills achieve. These seven teachers seemed to believe that contextualising maths gave a deeper understanding of the curriculum. They had considered the reason the curriculum is structured the way it is, which included the debate around numeracy versus calculator skills.

Theme 5: Teaching Strategies and Approaches

All twelve teachers (100%) were able to recall at least one strategy or approach they used to assist learners in maths. The strategies were diverse; however, the following categories cover most of the ideas that were mentioned in the interviews: engaging learning, explicit teaching, questioning and reflection. Seven teachers (58.3%) mentioned using some method to increase engagement. Four teachers (33.3%) stated that they use either worked examples or lecturing. Three teachers (25%) spoke of questioning strategies they had used. While three teachers had spoken of some form of provoking reflection on work.

Engaging learning took many forms. From the novel approaches of contextualising maths from Theme 4, to integrating technology into lessons, to how Participant L10 spoke of gamifying activities to make them more engaging (see appendix 3). It was noticeable that the body language and tone of a majority participants demonstrated enthusiasm for their engaging learning strategies.

Participant H8 said, “Worked examples are so powerful” and “I often teach like a lecturer.” Explicit teaching and demonstrating expert methods were supported by four of the teachers (25%), although only one (8.3%) of these teachers referred to the pedagogical theory vocabulary supporting this implying that they were the only teacher who was familiar with the research supporting their practice. The other teachers seemed unaware of the literature associated with their methods.

Questioning strategies were considered, three teachers demonstrated that they had developed questioning styles that were personal to their teaching style, yet also carefully formulated to elicit the desired level of thought. The questioning strategies revolved around trying to get the students to think about the overall problem-solving strategy, as opposed to procedural skills. This is seen by Participant I9's response "I ask them 'Why did we do that step?'. I want to know their thoughts; I want to know if they get why that step is important." Here the focus of the question is on the reasoning behind completing this step, not the skills necessary to carry out that step.

Finally, provoking some self-reflection over work was evident for a small number of participants (25%). Participant F6 demonstrated this most clearly when declaring, "At the end of an activity, I sometimes take time to get students to read back over their work and see if they can improve their method." The participant further elaborated "It's all about critical thinking, they have to analyse their approach." The two other participants briefly mentioned reflection as a useful learning strategy, with one of them suggesting that reflection develops them into "independent mature learners".

This concludes the finding section, in which the pertinent observations from interviews were reported. This section was non-exhaustive, and there were other noteworthy insights gathered (see Appendix 3). In the following section, the findings will be analysed and critiqued with reference to the literature review.

Discussion

Upon careful analysis of the findings compared to the literature review, it is now possible to reflect on the research aims and offer some answers. Throughout this section the findings sections will be critiqued with reference to the previously published research discussed in the literature review.

Theme 1: Maths Anxiety

A large majority of the participants spoke of their concerns around how emotions pose a barrier to effective problem-solving in students. The emphasis placed on this theme by the teachers was a striking contrast to the literature on mathematical pedagogy and problem-solving. Beilock and Willingham (2014) predicted emotional barriers were important to overcome in maths education, however this was a rarity in the literature. When reading the literature in the field of mathematical problem-solving pedagogy, it was not possible to foresee the extent to which teachers emphasised the roll of nurturing and emotional support in progress in mathematics. Perhaps a reason for the discrepancy is that the cognitive psychologists who tend to pursue this area of research are focussed on the acquisition of problem-solving skills in ideal situations (McEldoon et. al, 2013: Clark, Kirschner and Sweller, 2006: Alzahrani, 2017), and assume that the emotions issues have been addressed separately.

The teachers suggested that making maths more accessible would, and did, ease maths anxiety: through scaffolding or simplifying wording of questions. Teachers also spoke of how their nurturing and pastoral support can lower maths anxiety, motivating students to be more open to challenging mathematical scenarios. This later idea is consistent with Beilock and Willingham (2014) who suggested that teachers' emotionally supporting their students is an effective pedagogical intervention. Similarly, Beilock and Willingham (2014) did speak of adapting assessments to be more accessible, which is consistent with simplifying wording. However, scaffolding was not mentioned by Beilock and Willingham (2014). While both theorist and practitioner did acknowledge that assessments induce anxiety, Beilock and Willingham (2014) suggested that reducing the time pressure of exams would help, whereas the teachers did not mention time pressure, instead pointing to lengthy questions as the daunting aspect to assessments. Overall Beilock and Willingham's (2014) ideas have been largely substantiated by these present findings, with relatively minor details differing between their ideas and the opinions of the participants.

Theme 2: Comprehension as a Barrier to Problem-Solving

Most of the teachers mentioned comprehension as a barrier to mathematical problem-solving, with half of the participants describing this extensively. It was evident that comprehension was a significant concern of British-curriculum maths teachers. This issue was absent in the salient empirical research on the subject of mathematical problem-solving pedagogy (Hajar and In'am, 2017: Clark, Kirschner and Sweller, 2012: Centre for Education Statistics and Evaluation, 2017: Catrambone and Eiriksdottir, 2011), representing a concerning disconnect between the reality of teachers and the literature that is supposed to inform and support their practise. Upon further investigation and research of the literature in light of this finding, some small-scale studies did investigate links between comprehension and mathematical problem-solving ability. However, these studies merely identified the problem instead of suggesting specific pedagogical interventions for this problem (Wiest, 2003: Ulu, 2017: Gomez et al, 2020). The lack of current research on British-curriculum mathematical problem comprehension for secondary students acerbates the problem as teachers remain unsupported by theory and evidence as they try to counter this major difficulty experienced by students.

The teachers described the various strategies they had developed to support their students with comprehension: highlighting command words, acronyms and revealing sentences one by one. These strategies are a starting point for future research to validate or criticise and should be thoroughly investigated properly and empirically.

Theme 3: The Role of the Teacher

The teachers who participated in the interviews referred to their role as ‘lecturer’, ‘facilitator’, ‘mentor’, ‘advisor’ or even ‘head of the advisory board’. The terms lecturer and facilitator echo the debate in the literature around teacher-centred versus student-centred learning or explicit teaching versus discovery-based learning. While ‘mentor’ echoes the ideas expressed by Beilock and Willingham (2014). The word advisor appears novel in the context of mathematical pedagogy; however, it could be compared to the facilitator terminology from literature.

The balance of teachers favouring discovery-based learning to teachers favouring explicit teaching was tilted, by a slim margin, towards more teachers explicitly delivering worked examples. This reflects the debate in the literature base, with considerable support for discovery-based learning (Hajar and In’am, 2017: Lessani et al, 2016: Chang et al, 2010: Herdiana, Sispiyati and Wahyudin, 2017) being disputed by well-respected academics in the field of mathematical pedagogy who argue that explicit teaching is more effective (Clark, Kirschner anc Sweller, 2006: Clark, Kirschner and Sweller, 2012: Centre for Education Statistics and Evaluation, 2017, Crissman, 2006).

Interestingly, the teachers expressed new supporting ideas for discovery-based learning. One compared the discovery-learning process to the scientific method, and praised the opportunity afforded by the discovery process for students to spot patterns. This pattern spotting is reminiscent of the constructivism learning philosophy first described by Bruner (1961). The scientific method is how all new knowledge is formed, and therefore it is important for young people to develop an appreciation for the rigor in which the information delivered to them in school is formed. The teachers were able to defend their choices for discovery-based learning in a convincing way that had not been addressed by the sceptics, especially those sceptics who referred to discover-based learning as an ‘ideological’ pedagogical decision (Clark, Kirschner and Sweller, 2012).

Theme 4: Contextualising Maths

A small majority of participants spoke of the need to contextualise maths, to link the abstract subject with the real world and make it applicable. The interview findings indicate that the lack of understanding by students of how maths can be useful lead to the students losing motivation to learn. This was unmentioned in the salient literature on problem-solving pedagogy, probably because it falls into the category of student motivation as opposed to acquiring mathematical skills. However, the fact that a majority of teachers still brought this issue up shows that motivation remains intrinsically linked to problem-solving success in practice. Future research on this would be welcome, exploring whether contextualised mathematical problems which relate in an obvious way to a real-world application are better received by students than abstract problems without any clear applications.

The teachers disagreed with each other on the use of calculators versus mental arithmetic and traditional working out. The arguments presented by teachers were interesting and demonstrated the level of critical thought teachers practice in the profession. This was comparable to the ideas discussed in the literature review of encouraging rote learning in maths (Mhlolo, 2015: Narayan, 2009). Rote learning lowers the cognitive load on a student if they already know automatically basic number facts without having to interrupt their current train of thought (Centre for Education Statistics and Evaluation, 2017). Similarly, using a calculator automates parts of the problem-solving process, presumably lowering the cognitive load. However, a small number of participants saw calculators as a way to bypass the vital conceptual understanding of numbers that enables students to develop mastery of mathematics.

Theme 5: Teaching Strategies and Approaches

As seen in the findings section, teachers have developed various strategies to teach secondary British-curriculum maths. These strategies broadly belong to one of the following categories: engaging learning, explicit teaching, questioning and reflection.

The engaging learning that the participants described was generally more student-focussed than other teaching strategies. Engaging learning is synonymous with motivation. Motivation was one of the arguments in favour of student focussed discovery-based learning in the literature (Kistian, Armanto and Sudrajat, 2017), yet motivation was not addressed in the previous literature supporting cognitive load theory and explicit teaching in mathematics (Clark, Kirschner and Sweller, 2006: Clark, Kirschner and Sweller, 2012: Crissman, 2006: Bokosmaty, Sweller and Kalyuga, 2015). However, upon further inspection of the literature in light of these findings, Martin (2016) proposes that managing cognitive load through clear, structured and well-guided instruction leads to motivation and engagement. However, Martin (2016) acknowledges that the evidence base for this is lacking empirical data. Based on the findings of this present study, it appears that more teachers agree with Kistian, Armando and Sudrajat (2017), that student-centred teaching of problem-solving in mathematics is more engaging.

A minority of teachers explicitly supported teacher led worked examples. This is surprising considering the amount of support for explicit teaching from the most noteworthy academics in the field of cognitive psychology who focus on mathematical pedagogy. It is also noteworthy that only one participant used vocabulary derived from literature. It appears that the researchers must work harder if they are going to communicate their findings to practitioners and convince teachers to adjust their practice to align with the evidence base, as currently they are either unaware or unconvinced by the research.

The questioning and reflection strategies that the participants described lead students to consider the reasoning behind problem-solving steps. This is consistent with the ideas of metacognition (Izzati and Mahmudi, 2018: Alzahrani, 2017) and the subgoal-learning model (Catrambone and Eiriksdottir, 2011), which also aim to actualise the thought process behind effective problem-solving strategies. However, the questioning strategies that the teachers described allowed more engagement from the students, while lowering the control of the expert teacher to guide the reflection. This is an interesting distinction, as the subgoal learning model would have the teacher annotate the steps, the teachers instead asked the students to describe the steps.

Conclusion and Recommendations

This study sought to explore the lived experiences and perceptions of British curriculum secondary maths teachers on teaching problem-solving skills in practice. Particular emphasis was placed on comparing the similarities and differences in the practice of teachers to the ideas which populate the previous theory and literature. Furthermore, this study aimed to uncover divides between theory and practise such that they better complement each other.

The findings revealed similarities and differences between theory and practise. The most pertinent finding revealed the extent to which maths teachers see maths anxiety as a major barrier to be overcome if students are going to reach their potential as problem-solvers. The literature did not reflect this concern proportionately in line with the teachers' emphasis. Similarly, comprehension of mathematical text and questions was not addressed by the leading publications on mathematical problem-solving. Teachers expressed this clearly and repeatedly, pointing out that many exam questions are too complicated due to the wording and readability. Students cannot answer questions that they cannot read. Teachers have developed their own strategies to make these wordy exam questions more accessible to learners, and future research should explore these innovations to see if they are as effective as the teachers perceive them to be. Furthermore, future research on explicit teaching, discovery-based learning, cognitive load theory, metacognition or the sub-goal learning model should consider how these pedagogical strategies affect students' maths anxiety and question comprehension. If the literature continues to ignore the major concerns of maths teachers, it is unsurprising that maths teachers will continue to ignore the literature. This disconnect offers an explanation for a frustration expressed by two participants in the interviews; that their schools continued professional development was weak and unconvincing. This is perhaps another symptom of educational research being far removed from practice.

The debate on discovery-based learning versus explicit teaching continues in the literature and in practice. The participants were divided, like the literature, with some favouring student-centred learning while others defended the use of teacher-centred learning. The teachers introduced new arguments in favour of discovery-based learning, which were not properly rebuted by theorists who dismissed discovery-based learning as ‘ideological bias’ (Clark, Kirschner & Sweller, 2006; Clark, Kirschner & Sweller, 2012). The findings of this study imply that the debate is ongoing, not as settled as some would suggest. Indeed, many of the arguments in favour of discovery-based learning were echoed by the practitioners: engagement, motivation, independence and uncovering generalisations or patterns (Kistian, Armanto and Sudrajat, 2017: Herdiana, Sispiyati & Wahyudin, 2017).

Overall, the findings of this study demonstrate a disconnect between practitioners and researchers on several fronts, rarely expressing perfectly overlapping views, ideas or approaches. Several reasons are proposed to explain this discrepancy: research that does not address the priorities or concerns of teachers, research being separated from realistic classroom situations, research focussing on singular ideas without considering the many factors at play that influence learning and teachers not receiving effective evidence-based subject-specific professional development after initial training.

The main limitations of this present study are two-fold. This study is small-scale, with a cohort of twelve participants being too small to generalise the findings accurately, effectively and reliably to the thousands of British curriculum maths teachers worldwide.

Perhaps this limitation could be addressed with future research that took the findings from this study and aimed to validate or invalidate these findings with a larger scale survey which asked thousands of teachers to express their agreeance with the findings. This could possibly be achieved with a Likert scale design (Joshi et al, 2015). Furthermore, this study only gathered data on the subjective perceptions of teachers from their own perspective. It did not seek to substantiate their views with data on academic progress, student opinions of learning, views from other stakeholders in education or other data which could reinforce the subjective opinions of teachers. As the teachers expressed some ideas that were novel in the context of mathematical problem-solving, future research should explore these novel ideas from various research philosophies and approaches.

The world is full of unsolved problems. Climate change, the cure to cancer, steps forward in computer science, advances in engineering and problems that have not even been formulated yet are some of the real problems that the next generation of mathematicians will have to contend with. Their ability to contribute to advancements in human progress will partly depend on their teacher's ability to train them to become adept independent problem-solvers. For this reason, this study has made a small contribution to the field of mathematical pedagogy in the hopes that practitioners will benefit from more appropriate pedagogical strategies derived from empirical evidence in future studies. Ideally responding directly to the concerns and priorities expressed by maths teachers in this study who are on the front line of the profession.

References

Adams, D. McLaren, B. Mayer, R. Goguadze. & Isotani, S. (n.d) Erroneous Examples as Desirable Difficulty. Available from: https://core.ac.uk/reader/191337616

Adams, W. (2017) Conducting Semi-Structured Interviews. *George Washington University.* Available from: https://www.researchgate.net/publication/301738442_Conducting_Semi-Structured_Interviews

Anthony, G. & Whyte, J. (2012) Maths anxiety: The fear factor in the mathematics classroom. *New Zealand Journal of Teachers' Work.* Available from: https://www.researchgate.net/publication/260710496_Maths_anxiety_The_fear_factor_in_the_mathematics_classroom

Alzahrani, K. (2017) Metacognition and Its Role in Mathematics Learning: an Exploration of the Perceptions of a Teacher and Students in a Secondary School. Taif University. Available from: https://www.iejme.com/download/metacognition-and-its-role-in-mathematics-learning-an-exploration-of-the-perceptions-of-a-teacher.pdf

Barni, D. Francesca, D. & Benevene, P. (2019) Teachers' Self-Efficacy: The Role of Personal Values and Motivations for Teaching. Available from: https://www.researchgate.net/publication/334434447_Teachers%27_Self-Efficacy_The_Role_of_Personal_Values_and_Motivations_for_Teaching

Atkinson, R. Derry, S. Renkl, A. & Wortham, D (2000) Learning from Examples: Instructional Principles from the Worked Examples Research. Available from: https://journals.sagepub.com/doi/10.3102/00346543070002181

Ayres, P. Kalyuga, S. & Sweller, J. (2003) The Expertise Reversal Effect. Available from: https://www.researchgate.net/publication/48829036_The_Expertise_Reversal_Effect

Bailen, N. Green, L. & Thompson, R. (2018) Understanding Emotion in Adolescents: A Review of Emotional Frequency, Intensity, Instability, and Clarity. *Emotional Review.* Available from: https://cpb-us-w2.wpmucdn.com/sites.wustl.edu/dist/f/1305/files/2018/05/Bailen-Adolescent-Review-13h1n40.pdf

Barton, C. (2018) How I Wish I'd Taught Maths. John Catt Publications.

Barni, D. Francesca, D. & Benevene, P. (2019) Teachers' Self-Efficacy: The Role of Personal Values and Motivations for Teaching. Available from: https://www.researchgate.net/publication/334434447_Teachers%27_Self-Efficacy_The_Role_of_Personal_Values_and_Motivations_for_Teaching

Bates, B. (2019) Learning Theories Simplified. *SAGE Pub.* Available from: https://us.sagepub.com/sites/default/files/upm-assets/98828_book_item_98828.pdf

Beilock, S. & Willingham, D. (2014) Maths Anxiety: Can Teachers Help Students Recude It? *American Educator*. Available from:

http://mrbartonmaths.com/resourcesnew/8.%20Research/Anxiety/Maths%20Anxiety%20-%20Willingham.pdf

BERA (2018) Ethical Guidelines for Educational Research, fourth edition. Available from: https://www.bera.ac.uk/publication/ethical-guidelines-for-educational-research-2018-online

Bisra, K. Liu, Q. Nesbit, J. Salami, F. & Winne, P. (2018) Inducing Self-Explanation: a Meta-Analysis. *Educational Psychology Review.* Available from: https://link.springer.com/article/10.1007/s10648-018-9434-x

Bjork, E. & Bjork, R. (2009) Making Things Hard on Yourself, But in a Good Way: Creating Desirable Difficulties to Enhance Learning. https://bjorklab.psych.ucla.edu/wp-content/uploads/sites/13/2016/04/Ebjork_Rbjork_2011.pdf

Bokosmaty, Sweller & Kalyuga. (2015) Learning Geometry Problem Solving by Worked Examples: Effects on Learner Guidance and Expertise. *American Educational Research Journal.* Available from : https://0-www-jstor-org.serlib0.essex.ac.uk/stable/pdf/24546756.pdf?refreqid=excelsior%3Aab68b6ec33b5c6c5393850cb12fe2e44

Brown, D. & Roy, Ranu. (2013) Will the Mathematics Teaching Methods Used in Eastern Countries Benefit Western Learners? *British Journal of Education, Society & Behavioural Science.* Available from: https://www.journaljesbs.com/index.php/JESBS/article/view/17110

Bruner, J. S. (1961). The act of discovery. *Harvard Educational Review*, 31, 21-32.

Catrambone, R. (1998) The Subgoal Learning Model: Creating Better Examples So That Students Can Solve Novel Problems.

Catrambone, R. & Eiriksdottir, E. (2011) Procedural Instructions, Principles, and Examples: How to Structure Instructions for Procedural Tasks to Enhance Performance, Learning, and Transfer. *SAGE Publications.* Available from : https://www.researchgate.net/publication/221738861_Procedural_Instructions_Principles_and_Examples_How_to_Structure_Instructions_for_Procedural_Tasks_to_Enhance_Performance_Learning_and_Transfer

Centre for Education, Statistics and Evaluation. (2017) Cognitive load theory: Research that teachers really need to understand. Available from: https://www.cese.nsw.gov.au//images/stories/PDF/cognitive-load-theory-VR_AA3.pdf

Chamberlin, S. (2008) What is Problem Solving in the Mathematics Classroom? Available from: http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome23/Chamberlin%20What%20is%20Math%20Prob%20Solving.doc

Chang, T. Chan, T. Liao, C. & Yang, F. (2010) The Effectiveness of Inductive Discovery Learning in 1: 1 Mathematics Classroom. Available from: https://www.semanticscholar.org/paper/The-Effectiveness-of-Inductive-Discovery-Learning-1-Yang-Liao/31fddd6b3921dac13a2e0e0c60b39d8a99cf18cd

Chapman, O. (2015) Mathematics Teachers' Knowledge for Teaching Problem Solving *University of Calgary*. Available from: https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=&ved=2ahUKEwjHq7KA4YzvAhUlShUIHWEOCpIQFjABegQIBhAD&url=https%3A%2F%2Fjournals.helsinki.fi%2Flumat%2Farticle%2Fdownload%2F1049%2F1042&usg=AOvVaw3zGhn2JdJsjDX3jB4QhHlT

Chi, M. (1998) Self-explaining: The dual processes of generating inferences and repairing mental models. Learning Research and Development Center. Available from: https://education.asu.edu/sites/default/files/advances_3.pdf

Clark, N. (2014) The Major International School Curriculums. *World Education News & Reviews.* Available from: https://wenr.wes.org/2014/07/the-major-international-school-curriculums

Clarke, R. Kirschner, P. & Sweller, J. (2012) Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching. Educational Psychologist. Available from: https://www.tandfonline.com/doi/pdf/10.1207/s15326985ep4102_1

Clarke, R. Kirschner, P. & Sweller, J. (2012) Putting Students on the Path to Learning, The Case for Fully Guided Instruction. *American Educator.* Available from: https://www.aft.org/sites/default/files/periodicals/GuidedInstruction.pdf

Clarke, V., Braun, V., & Hayfield, N. (2015). Thematic analysis. Qualitative psychology: A practical guide to research methods, 222-248.Cohen, L., Manion, L., & Morrison, K. (2018) Research Methods in Education. (8thed). New York: Routledge.

Cohen, L. Manion, L. & Morrison, K. (2018) Research methods in education. *Routledge.* Available from: https://www.worldcat.org/title/research-methods-in-education/oclc/1015339163

Crissman, J. (2006) The design and utilization of effective worked examples: A meta - analysis. *University of Nebraska.* Available from: https://www.researchgate.net/publication/282228422_The_design_and_utilization_of_effective_worked_examples_A_meta_-analysis

Department of Education, UK. (2020) Initial teacher training (ITT): criteria and supporting advice. Available from: https://www.gov.uk/government/publications/initial-teacher-training-criteria/initial-teacher-training-itt-criteria-and-supporting-advice

Department of Psychology, Harvard. (n.d) Strategies for Qualitative Interview. Available from: https://sociology.fas.harvard.edu/files/sociology/files/interview_strategies.pdf

Downs, M. & Mamona-Downs, J. (2005) The identity of problem solving. *Journal of Mathematical Behavior.* Available from: https://www.researchgate.net/publication/257238610_The_identity_of_problem_solving

Durkin, K. (2011) The Self-Explanation Effect When Learning Mathematics: A Meta-Analysis. Available from: https://files.eric.ed.gov/fulltext/ED518041.pdf

Edexcel. (2014) A Guide to Edexcel GCSE Mathematics (9-1). Available from: https://qualifications.pearson.com/content/dam/pdf/GCSE/mathematics/2015/misc/T772%20GCSE%20Maths%20Guide%20(web%20-%20final).pdf

Educational Support, 2019. Teacher Wellbeing Index 2019. Available from: https://www.educationsupport.org.uk/sites/default/files/teacher_wellbeing_index_2019.pdf

Eisenhart, C. & Wei, M. (2011) Why do Taiwanese children excel at math? *Global Voices.* Available from: https://www.academia.edu/987689/Why_do_Taiwanese_Children_Excel_at_Math

Flahive, M. & Kasman, R. Guided discovery in a discrete mathematics course for middle school teachers. Available from: http://sites.science.oregonstate.edu/~flahivem/PDFolder/PaperFolder/reva.pdf

Fusch, P. I., & Ness, L. R. (2015). Are we there yet? Data saturation in qualitative research. The qualitative report. Available from: https://nsuworks.nova.edu/tqr/vol20/iss9/3/

Giganti, P. (2007) Why Teach Problem Solving, Part I: The World Needs Good Problem Solvers!
CMC Maths. Available from: https://www.cmc-math.org/assets/docs/WhyTeachProblemSolvingPart1.pdf

Given, L. (2008) Lived Experience. Available from: https://methods.sagepub.com/reference/sage-encyc-qualitative-research-methods/n250.xml

Gomez, A. Pecina, E. Villanueva, S. & Huber, T. (2020) The undeniable relationship between reading comprehension and mathematics performance. Available from: https://www.iier.org.au/iier30/gomez.pdf

Hajar, S. & In'am, A. (2017) Learning Geometry through Discovery Learning Using a Scientific Approach. *International Journal of Instruction.* Available from: https://www.researchgate.net/publication/313419459_Learning_Geometry_through_Discovery_Learning_Using_a_Scientific_Approach

Hatry, H. Newcomer, K. & Wholey, J. (2015) Handbook of Practical Program Evaulation. *Jossey-Bass.* Available from: https://www.researchgate.net/publication/301738442_Conducting_Semi-Structured_Interviews 500-504

Herdiana, Y. Sispiyati, D. & Wahyudin, W. Effectiveness of Discovery Learning Model on Mathematical Problem Solving. *Department of Mathematics Education, Indonesia University of Education.* Available from: https://aip.scitation.org/doi/pdf/10.1063/1.4995155

Hervey, S. & Klerein, J. (2018) Mathematics as a Complex Problem-Solving Activity: Promoting students' thinking through problem-solving. Available from: https://36kf1rfh5v23ru41h2gm84pz-wpengine.netdna-ssl.com/wp-content/uploads/2019/02/Mathematics-as-a-Complex-Problem-Solving-Activity.pdf

Izzati, L. & Mahmudi, A. (2018) The Influence of Metacognition in Mathematical Problem Solving. Available from: https://iopscience.iop.org/article/10.1088/1742-6596/1097/1/012107#:~:text=Metacognition%20is%20an%20important%20factor,defined%20as%20think%20about%2
0thinking.

Joffe, J. (2007) Scientific Psychology: Introduction to Research Methods and Research Designs. Available from: https://www.researchgate.net/publication/303738651_Scientific_Psychology_Introduction_to_Research_Methods_and_Research_Designs_-_Research_Methodology

Johnson, T. (2014) Snowball Sampling: Introduction. Available from: https://www.researchgate.net/publication/316414208_Snowball_Sampling_Introduction

Joshi, A. Kale, S. Chandel, S. & Pal, D. (2015) Likert Scale: Explored and Explained. Available from: https://www.researchgate.net/publication/276394797_Likert_Scale_Explored_and_Explained

Klerlein, J. & Hervey, S. (2018) Mathematics as a Complex Problem-Solving Activity. Available from: https://36kf1rfh5v23ru41h2gm84pz-wpengine.netdna-ssl.com/wp-content/uploads/2019/02/Mathematics-as-a-Complex-Problem-Solving-Activity.pdf

Kirchner, P. Sweller, J. & Clark, R. (2006) Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist, Discovery, Problem-Based, Experiential, and Inquiry Based Teaching. Available from: https://www.tandfonline.com/doi/abs/10.1207/s15326985ep4102_1

Kistian, A. Armanto, D. & Sudrajat, A. (2017) The Effect Of Discovery Learning Method On The Maths Learning Of The V SDN 18 Students Of Banda Aceh, Indonesia. *British Journal of Education.* Available from: http://www.eajournals.org/wp-content/uploads/The-Effect-of-Discovery-Learning-Method-on-the-Math-Learning-of-the-V-Sdn-18-Students-of-Banda-Aceh-Indonesia.pdf

Leaderbeater, C. (2016) The Problem Solvers. Available from: https://www.pearson.com/content/dam/corporate/global/pearson-dot-com/files/learning/Problem-Solvers-Web-.pdf

Lessani, A. Yunus, A. Bakar, K. & Khameneh, Z. (2016) Comparison of Learning Theories in Mathematics Teaching Methods. Available from: http://archives.evergreen.edu/masterstheses/Accession89-10MIT/Solso_DMITthesis2009.pdf

Lee, S. (1999) Problem Solving in a Constructivist Environment. Available from: https://www.researchgate.net/publication/26391074_Problem-solving_in_a_Constructivist_Environment/link/564d0e6f08aefe619b0dc5bb/download

Lincoln, Y. S., & Guba, E. G. (1985). Naturalistic inquiry. Beverly Hills, CA: Sage.

Maguire, M., & Delahunt, B. (2017). Doing a thematic analysis: A practical, step-by-step guide for learning and teaching scholars. *All Ireland Journal of Higher Education.*

Margulieux, L. (2014) Subgoal Labeled Instructional Text and Worked Examples in STEM Education. *Georgia Institute of Technology.* Available from: https://smartech.gatech.edu/bitstream/handle/1853/51782/MARGULIEUX-THESIS-2014.pdf?sequence=1&isAllowed=y

Martin, A. (2016) Using Load Reduction Instruction (LRI) to boost motivation and engagement. *University of New South Wales, Australia.* Available from: https://www.researchgate.net/publication/310102750_Using_Load_Reduction_Instruction_LRI_to_boost_motivation_and_engagement

McEldoon, K. Durkin, K. & Johnson, B. (2013) Is self-explanation worth the time? A comparison to additional practice. Available from: https://pubmed.ncbi.nlm.nih.gov/24175685/

McGrath, C. Palmgren, P. & Liljedahl, M. (2018) Twelve tips for conducting qualitative research interviews. *Medical Teacher.* https://www.tandfonline.com/doi/full/10.1080/0142159X.2018.1497149

McLeod, S. A. (2017). Behaviorist approach. Available from: www.simplypsychology.org/behaviorism.html

Merrienboer, J. & Paas, F. (1998) Cognitive Architecture and Instructional Design. Education Psychology Review. Available from: https://www.researchgate.net/publication/200772805_Cognitive_Architecture_and_Instructional_Design

Mujis, D. Quigley, A. & Stringer, E. (2018) Metacognition and Self-Regulated Learning. Available from: https://educationendowmentfoundation.org.uk/public/files/Publications/Metacognition/EEF_Metacognition_and_
self-regulated_learning.pdf

Mhlolo, M. (2015) Is Rote Learning of Number Concepts 'Inherently Rotten' or Is It Just a Blame and Shame Game that Vitiates Principles of Natural Progression? Mediterranean Journal of Social Sciences. Available from: https://www.researchgate.net/publication/281585954_Is_Rote_Learning_of_Number_Concepts_'Inherently_Rotten'_or_Is_It_Just_a_Blame_and_Shame_Game_that_Vitiates_Principles_of_Natural_Progression_Michael_Kainose_Mhlolo_Dr

Narayan, A. (2009) Rote and Algorithmic Techniques in Primary Level Mathematics Teaching in the Light of Gagne's Hierarchy. Available from: https://pdfs.semanticscholar.org/a9b3/77229ad9ebcc12a27900badb64f95322b03d.pdf

National Council of Teachers of Mathematics, n.d. Procedural Fluency in Mathematics. Available from: https://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics/

National Research Council. (2001) Adding It Up: Helping Children Learn Mathematics. Page 121

Opie, C. & Browm, D. (2019) Getting Started in your Educational Research.

Ozdamli, F. & Asiksoy, G. (2016). Flipped classroom approach. *World Journal on Educational Technology: Current Issues*. Available from: https://www.researchgate.net/publication/309890120_Flipped_Classroom_Approach

Pharm, J. (2014) Qualitative research method-interviewing and observation. Available from: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4194943/

Polya, G. (1945). How to solve it; a new aspect of mathematical method. Princeton University Press.

PWC. (2018) Will robots really steal our jobs? An international analysis of the potential long-term impact of automation. Available from: https://www.pwc.fr/fr/assets/files/pdf/2018/02/impact-of-automation-on-jobs-international-analysis-final%20report-feb-2018.pdf

PWC. (n.d) 10 Skills You Need for Future Employment. Available from: https://www.pwc.com.au/careers/blog/future-employment.html

Ryan, F. Coughlan, M. & Cronin, P. (2009) Interviewing in qualitative research. Available from: https://www.researchgate.net/publication/261471599_Interviewing_in_qualitative_research

Schunk, D. (2012) Learning Theories: An Educational Perspective. *Pearson.* Available from: http://repository.umpwr.ac.id:8080/bitstream/handle/123456789/96/%5bDale_H._Schunk%5d_Learning_Theories_An_Educational..pdf?sequence=1

Schoenfeld, A. (1992) Learning to Think Mathematically: Problem Solving, Metacognition and Sense-Making In Mathematics.

Star, J. & Johnson, B. (2008) Flexibility in problem solving: The case of equation solving. Available from: https://www.researchgate.net/publication/222433916_Flexibility_in_problem_solving_The_case_of_equation_s
Olving

Suarta, M. Suwintana, K. Pranadi, F. & Hariyanti, N. (2017) Employability Skills Required by the 21st Century Workplace: A Literature Review of Labor Market Demand. Available from: https://www.researchgate.net/publication/320469836_Employability_Skills_Required_by_the_21st_Century_Workplace_A_Literature_Review_of_Labor_Market_Demand

Svabo, Z. Körtesi, P. Guncaga, J. Szabo, D. & Neag, R. (2020) Examples of Problem-Solving Strategies in Mathematics Education Supporting the Sustainability of 21st-Century Skills. Available from: https://www.mdpi.com/2071-1050/12/23/10113/pdf

Sweller, J. (1988) Cognitive Load During Problem Solving: Effects on Learning. Available from: https://onlinelibrary.wiley.com/doi/pdfdirect/10.1207/s15516709cog1202_4

Trafton, J. & Reiser, B. (n.d). The Contribution of Studying Examples and Solving Problems to Skill Acquisition
Available from:
https://www.academia.edu/270647/The_Contributions_of_Studying_Examples_and_Solving_Problems_to_Skill
_Acquisition

Tricot, A. & Sweller, J. (2014) Domain-Specific Knowledge and Why Teaching Generic Skills Does not Work.

UKMT. (2020) Available from:
https://www.ukmt.org.uk/sites/default/files/ukmt/juniormathematical-challenge/JMC-2020-q.pdf

Ulu, M. (2017) The Effect of Reading Comprehension and Problem Solving Strategies on Classifying Elementary 4th Grade Students with High and Low Problem Solving Success. Available from: https://files.eric.ed.gov/fulltext/EJ1141427.pdf

UNICEF. (2019) Global Framework on Transferable Skills. Available from:
https://www.unicef.org/media/64751/file/Global-framework-on-transferable-skills-2019.pdf

University of Essex. (2018) Guidelines for Ethical Approval of Research Involving Human Participants. Available from: https://www.essex.ac.uk/-/media/documents/directories/reo/university-guidelines-for-ethical-approval-december-2018.pdf

Wellington, J. (2015) Educational Research, Contemporary Issues and Practical Approaches. *Bloomsbury.*

White Rose Maths. (2016) Available from: http://www.southshieldsschool.co.uk/wp-content/uploads/2018/03/GCSE-Problem-Solving-Questions-Higher-and-Foundation.pdf

Wiest, L (2003) Comprehension of Mathematical Text. Available from:
http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome17/pdf/lwiest.pdf

Wilam, D. (2017) Available from:
https://twitter.com/dylanwiliam/status/824682504602943489

Yildiz, C. (2016) Investigation of Mathematics Teachers' Views About Improving Problem Solving. Available from:
https://www.researchgate.net/publication/324796339_INVESTIGATION_OF_MATHEMATICS_TEACHERS'_VIEWS
_ABOUT_IMPROVING
_PROBLEM_SOLVING_SKILLS/link/5ae2c9b1a6fdcc9139a10104/download

Appendix 1 :Consent Letter :

Dear Participant,

Thank you volunteering to be part of this project. Please read the statement below and if you agree to take part in the research project, please sign and print your name at the bottom. If you have any questions, please don't hesitate to contact the researcher: David (Will) Mcloughlin.

The project will seek to understand the opinions and experiences of maths teachers. This will be conducted through 30 minute online video interviews. This is part of my Masters in Education dissertation with the University of Essex, UK. I (David) am a maths teacher currently living in Abu Dhabi, with a keen interest in mathematical pedagogy.

"I understand that this is a project pertaining to teaching problem solving skills in maths education and my own experiences on teaching these skills. There is no compulsion for me/my institution to take part in the research. If I chose not to take part, I will not be prejudiced in any way and the researchers will respect my decision as an educational professional. If I do choose to take part, I may at any stage withdraw my involvement. If I choose to withdraw, this will not affect my professional position. Any information given by me will be used solely for the purposes of this research, which may include publications. No individuals will be identified in the final publication. Confidentiality will be respected by the researcher in relation to the information which I give.

The interview will be recorded and stored securely, unless I request otherwise. The researcher can make accommodation if I feel uncomfortable with a interview recording, however this may slightly increase the time taken to conduct the interview.

Any data gathered during the course of the research will be stored securely on password protected devices and any personal data will be destroyed after the research has been published, however anonymised data may be published as part of the research project.

I have read and understood the nature of my involvement in the project and agree to take part in it.

By signing below I agree to the above statement, and understand that I can withdraw consent at any time throughout the duration of the research project."

Signed……………………………….

Print name…………………………………

Contact:
David (Will) Mcloughlin
dm19673@essex.ac.uk

Appendix 2: Interview Structure

	Question	Reasoning	Follow Up Question	Reference
1	*Thank you for your time today and sharing your experiences. Please tell me about your interest in maths education?*	To build a rapport between the interviewer and interviewee.	Why did you choose a career in maths education? What is your favourite part of your job?	Modified from McGrath, Palmgren and Liljedahl (2019)
2	*When you first started teaching maths, what did you find came naturally? What did you find more challenging?*	To ensure the participants are suitably experienced and qualified to offer reliable responses as professional maths teachers, while also further establishing a narrative of their professional experiences.	Can you give me some examples of why you think that? How come? If you could give advice to someone just beginning a career in teaching, what would you say?	Adapted from Hatry, Newcomer and Wholey (2015)
3	*Which skills do you think are the most important or transferable that are taught in maths?*	To contextualise the interview for the participant and show an interest in their professional views. To also gain perspective on the priorities of maths teachers.	Why have you identified that skill in particular? In your opinion, does the current curriculum give this area sufficient focus? I notice that you don't include problem-solving skills, why have you not mentioned these?	Opie and Brown (2019)
4	*What do you understand by the term 'Problem-solving skills'?*	Leads into ensuring the interviewer and interviewee are clear on the key definition.	For the remainder of this interview, the following definition will be used 'carrying out a logical process to arrive at an answer	Klerlein and Hervey (2018)

			to a novel and non-routine question'. What do you think of this definition?	
I am now going to move onto discussing problem-solving skills.		Structures the interview and focussed future questions.		Harvard, n.d.
5	*What do you find most challenging about teaching problem-solving skills?* *What do you find easiest about teaching problem-solving skills?*	Lays a foundation for future questions which will probe further into how the teachers overcame these hurdles. Remain flexible with question order, if the answer naturally leads into question	Why is that? How have you adapted to these challenges? What particularly did the students struggle with?	Harvard, n.d.
6	*Is there a particular instance of you teaching problem-solving skills in maths that you can remember?* *Can you describe this experience?*	Elicits personal reflection of relevant experiences.	What made you think of this instance? What was your role in this scenario? What went well? What inspired you to take this approach? What would you improve next time? How did the students respond to this lesson?	Opie and Brown (2019)
7	*Are there any particular teaching strategies that you consider most effective to teaching problem-solving?*	Leads to discussion around the pedagogical reasoning behind their teaching decisions.	Why do you find this strategy effective? What do you find difficult or about carrying out this teaching strategy? Are there any challenges that you have encountered?	Adapted from Wellington (2015)

8	*Interviewer shows 3 problem-solving questions on the Zoom screen. (See below)* *Here are 3 UK curriculum maths questions. Do you have any thoughts on them?*	Gives a visual prompt to promote further inciteful conversation.	How can teachers help students adapt to novel challenges like this? Do you find that students enjoy challenges like these? What do you think makes these particularly difficult? How could teachers make these questions more accessible for novice learners?	Opie and Brown (2019) White Rose Maths (2016) UKMT (2020)
9	*Is there anything else you would like to discuss relating to your experience of teaching problem-solving skills?*	Offers opportunity for participants to expand on their answers so far.		McGrath, Palmgren and Liljedahl (2019)

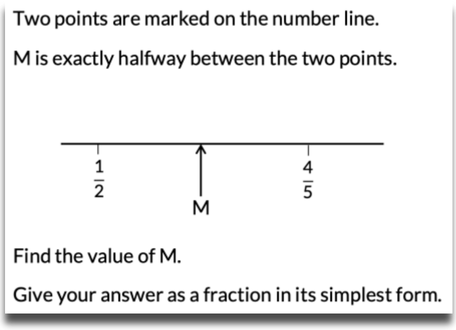
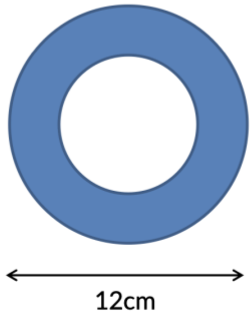


Figure 1: Problem-Solving Example 1 (White Rose Maths, 2016)

A shape is made of up two circles.



The diameter of the large circle is 12cm.

The area of the shaded region is 50cm^2

Find the diameter of the smaller inner circle.

Figure 2: Problem-Solving Example 2 (White Rose Maths, 2016)

7. Four different positive integers have a product of 110. What is the sum of the four integers?

A 19 B 22 C 24 D 25 E 28

Figure 3: Problem-Solving Example (UKMC, 2020)

Appendix 3: Thematic Analysis

Themes		Categories	Transcripts
1.	Maths Anxiety	Emotional. Barriers to problem-solving. Support. Anxious. Confidence. Growth mindset.	Participant B2: “They get flustered.”
			Participant C3: “It [maths anxiety] is a massive problem, it really stifles the kids.” “They don’t think clearly.”
			Participant D4: “They have a fear of failing.”
			Participant F6 “Pupils need nurturing when they have low confidence.”
			Participant G7: “I think the pastoral side of teaching can really help.” “Yes, I have seen it improve progress.”
			Participant I9: “Once they’ve done it, it doesn’t seem so bad.” “If you get something wrong, you keep going, you don’t let it break you. That’s problem-solving, it builds resilience, that’s a life skill.”
			Participant J10: “[students have a] fear of failure.”
2.	Comprehension as a barrier to Mathematical Problem-Solving	EAL. Questions posed by several long sentences. Leads to Maths anxiety. Give-up.	Participant A1: “They don’t know the basic [mathematical] vocabulary.”
			Participant C3 “The boys I teach are mostly EAL, it’s very difficult for them to understand the long-winded questions.”
			Participant E5 “If it looks complicated, they give-up.”
			Participant G7 “I teach a lot of EAL learners, they don’t like wordy questions. I have to spend a lot of time breaking down the sentences with them.”
			Participant H8: “Focus on the command words in the question.”
			Participant K11 “[when] pupils see questions in exams with long paragraphs, they just turn the page.” “They just don’t understand the question. It’s impossible to explain the maths to them if they don’t understand the context.”
			Participant L12 “I reveal the first sentence of the question, we discuss together. Once

			they are happy, we continue with the second sentence." "Highlight the command words."
3.	Role of the Teacher.	Student centred teaching. Teacher centred teaching. Explicit teaching. Lecture. Questioning. Independent work. Discovery based learning. Facilitator. Modelling expert solutions.	Participant B2: "[In this scenario] I was the head of the advisory board, the pupils had to convince me of their approach throughout the project."
			Participant C3 "I ask them what things we have learned could help them with this question. They come up with ideas. If their ideas are wrong, I still talk them through it, and try to show them why their ideas might not pan out."
			Participant D4: "Teach the how before the why." "The 'why' will come later."
			Participant E5: "The kids enjoy it [independent work], and it really helps when they spot the patterns themselves, they understand it better."
			Participant G7: "It can be a double-edged sword. Sometimes I have to model the expert way of doing things, but sometimes the students get scared to ask questions if I talk like that. They feel like it's not a safe space."
			Participant H8: "Only when they're confident will I give them unguided activities."
			Participant J10 "Lower classes struggle with independent work, I would only use it [discovery based learning] for top sets"
			Participant K11: "Lecturing gives me control."
			Participant L12: "I circulate around the classroom while they're working in groups, I observe mostly but sometimes ask them questions. This can guide them in the right direction."
4.	Contextualising Maths.	Calculator. Simplifies calculations. Estimation. Mental arithmetic.	Participant A1: "I encourage pupils to estimate an answer before trying it, it helps them check and reinforces the context of the question. For example, if they are working out the length of a side of a triangle, I make them guess it first. It helps them connect the numbers to the real world."

		Rote learning. Applying maths. Real-world scenarios.	Participant B2: “Fluency with basic maths skills definitely makes problem-solving easier.” “Arithmetic comes from everyday life, going to the shop or recipes.”
			Participant C3: “They always ask ‘where will we ever use this?’, they don’t understand why the maths is useful.”
			Participant E5: “If we wanted maths to be directly applicable to the real-world, we would teach tax returns before we taught abstract geometry, but we don’t.”
			Participant F6: “They often know individual steps, but don’t know how they all fit together.”
			Participant G7: “They will use a calculator for GCSE exams, so every lesson I make them use a calculator from Year 8 upwards.”
			Participant H8: “They were asked to work on an interior design project, they had to bring business, art and maths together.”
			Participant L12: “Sometimes we do different things, like once we baked a cake as a class to learn about circumference and area.” “Yes, it was engaging, I think they remembered it better when it was about the real world.”
			Participant J12: “Students just don’t have the foundation skills necessary to access the curriculum.”
5.	Teaching Strategies and Approaches.	Cross-curricular problem-solving. Activities. Visualising. Applications of problem-solving.	Participant A1: “I’d say its compartmentalised. They can’t bring two topics together. If a question involved algebra and geometry they don’t know where to start.” “I would always start with jogging prior knowledge, the knowledge that is prerequisite for that lesson.”

		Reflection. Engaging learning. Project based learning.	Participant B2: “I get the students to read over a model answer. I then ask questions about it; ‘What was the point of this step?’, ‘Why did we do that bit first?’ ‘Is there another way we could get to the correct answer?’”.
			Participant C3: “I guide them through an investigation, we use Desmos [online graphing software] to plot difference graphs and they try to spot the patterns, to see the rules. It’s tricky but when it works, it definitely works.” “It’s like they are doing science experiments in maths class, it feels like they are true mathematicians and scientists.”
			Participant D4: “We tasked the students with creating an interior design company. They had to advertise, plan, budget and pitch. It was a great lesson.” “Discovery [based] learning is engaging, it’s fun but can also be really beneficial when it becomes a pattern spotting activity.”
			Participant F6: “At the end of an activity, I sometimes take time to get students to read back over their work and see if they can improve their method.” “It’s all about critical thinking, they have to analyse their approach.”
			Participant H8: “Worked examples are so powerful.” “I often teach like a lecturer.” “Scaffolding helps. I give small hints, written or spoken, that suggest what steps they should take next.”
			Participant I9: “I ask them ‘Why did we do that step?’. I want to know their thoughts; I want to know if they get why that step is important.”

			Participant L10: “Students are very competitive, sometimes games or class competitions can get them involved.”
			Participant K11: “Exposure is key, they need to practice often.” “Flipped learning is useful, we use it at our school.”
			Participant L12: “Sometimes we do different things, like once we baked a cake as a class to learn about circumference and area.” “Yes, it was engaging, I think they remembered it better when it was about the real world.”