Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Specimen

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

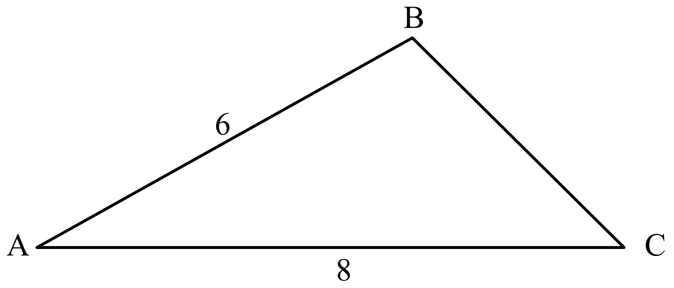
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

diagram not to scale



(a) Given that $\cos \hat{A} = \frac{5}{6}$, find the value of $\sin \hat{A}$. [3]

(b) Find the area of triangle ABC. [2]

2. [Maximum mark: 5]

Let A and B be events such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cup B) = 0.6$.
Find $\mathrm{P}(A \mid B)$.

3. [Maximum mark: 5]

(a) Show that $(2n-1)^2+(2n+1)^2=8n^2+2$, where $n\in\mathbb{Z}$. [2]

(b) Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even. [3]

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4. [Maximum mark: 5]

Let $f'(x) = \frac{8x}{\sqrt{2x^2 + 1}}$. Given that $f(0) = 5$, find $f(x)$.

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5. [Maximum mark: 5]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

6. [Maximum mark: 8]

(a) Show that $\log_9(\cos 2x + 2) = \log_3\sqrt{\cos 2x + 2}$. [3]

(b) Hence or otherwise solve $\log_3(2\sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. [5]

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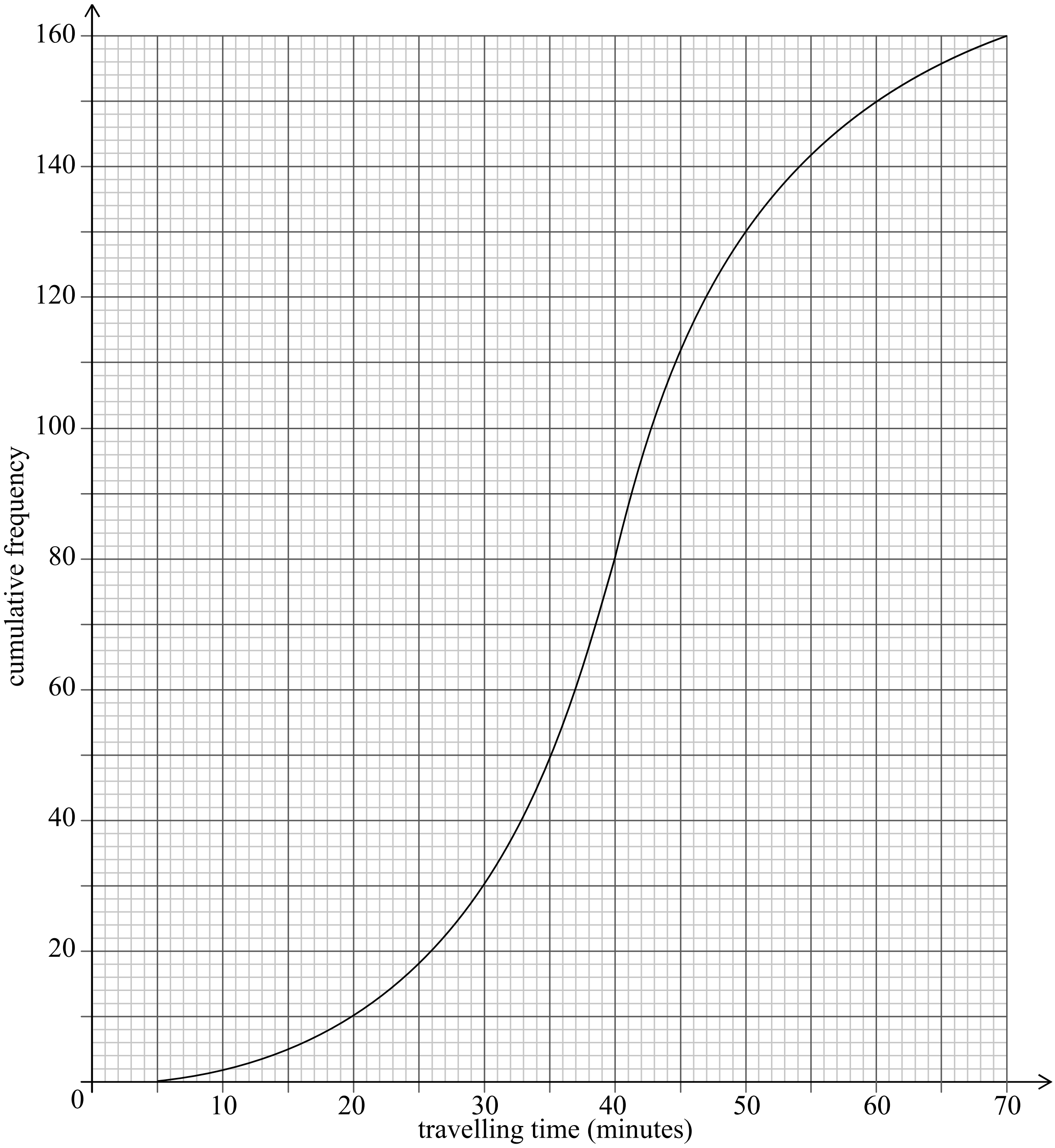
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

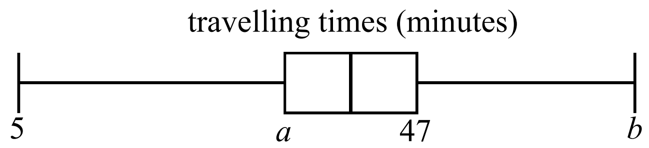
(a) Find the median number of minutes spent traveling to work. [2]

(b) Find the number of employees whose travelling time is within 15 minutes of the median. [3]

Only 10% of the employees spent more than k minutes traveling to work.

(c) Find the value of k. [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.



(d) Write down the value of b. [1]

(e) (i) Find the value of a.

(ii) Hence, find the interquartile range. [4]

Travelling times of less than p minutes are considered outliers.

(f) Find the value of p. [2]

8. [Maximum mark: 16]

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

(a) Find $f'(x)$. [2]

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(b) Find the value of a and the value of b. [3]

(c) (i) Sketch the graph of $y = f'(x)$.

(ii) Hence explain why the graph of f has a local maximum point at $x = a$. [2]

(d) (i) Find $f''(b)$.

(ii) Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$. [4]

The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q).

(e) Find the value of p and the value of q. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Let $f(x) = \dfrac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

(a) Show that $f'(x) = \dfrac{1 - \ln 5x}{kx^2}$. [3]

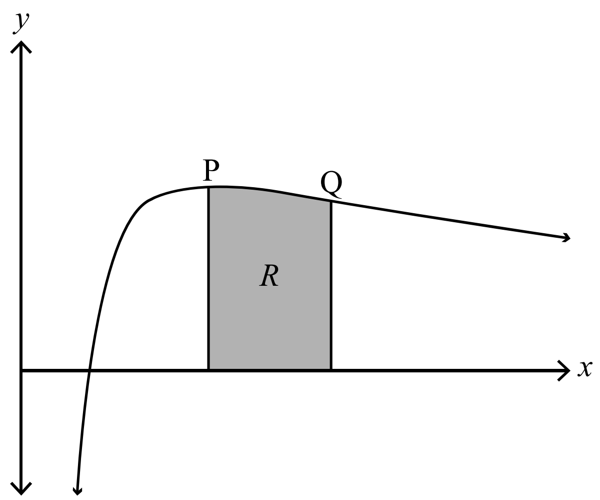
The graph of f has exactly one maximum point P.

(b) Find the x-coordinate of P. [3]

The second derivative of f is given by $f''(x) = \dfrac{2\ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

(c) Show that the x-coordinate of Q is $\dfrac{1}{5}\mathrm{e}^{\frac{3}{2}}$. [3]

The region R is enclosed by the graph of f, the x-axis, and the vertical lines through the maximum point P and the point of inflexion Q.



(d) Given that the area of R is 3, find the value of k. [7]

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will not be marked.

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Answers written on this page will not be marked.

Markscheme

Specimen paper

Mathematics: analysis and approaches

Standard level

Paper 1

11 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then* ***FT*** *marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r>1$ for the sum of an infinite GP, $\sin\theta=1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta=1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

Section A

1. (a) valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

(b) $\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

$\text{area} = 4\sqrt{11}$ ***A1***

[2 marks]

Total [5 marks]

2. attempt to substitute into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ ***(M1)***

Note: Accept use of Venn diagram or other valid method.

$0.6 = 0.5 + 0.4 - \mathrm{P}(A \cap B)$ ***(A1)***

$\mathrm{P}(A \cap B) = 0.3$ (seen anywhere) ***A1***

attempt to substitute into $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ ***(M1)***

$= \frac{0.3}{0.4}$

$\mathrm{P}(A \mid B) = 0.75 \left(= \frac{3}{4}\right)$ ***A1***

Total [5 marks]

3. (a) attempting to expand the LHS *(M1)*

$\text{LHS} = \left(4n^2 - 4n + 1\right) + \left(4n^2 + 4n + 1\right)$ ***A1***

$= 8n^2 + 2\left(= \text{RHS}\right)$ ***AG***

[2 marks]

(b) ***METHOD 1***

recognition that $2n-1$ and $2n+1$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$8n^2 + 2 = 2\left(4n^2 + 1\right)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

METHOD 2

recognition, *eg* that n and $n+2$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$n^2 + \left(n+2\right)^2 = 2\left(n^2 + 2n + 2\right)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

Total [5 marks]

4. attempt to integrate *(M1)*

$u = 2x^2 + 1 \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 4x$

$\displaystyle\int \frac{8x}{\sqrt{2x^2+1}}\,\mathrm{d}x = \int \frac{2}{\sqrt{u}}\,\mathrm{d}u$ ***(A1)***

EITHER

$= 4\sqrt{u}\left(+C\right)$ ***A1***

OR

$= 4\sqrt{2x^2+1}\left(+C\right)$ ***A1***

THEN

correct substitution into **their** integrated function (must have C) ***(M1)***

$5 = 4 + C \Rightarrow C = 1$

$f\left(x\right) = 4\sqrt{2x^2+1} + 1$ ***A1***

Total [5 marks]

5. (a) attempt to form composition *M1*

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ *A1*

$(g\circ f)(x)=2x+11$ *AG*

[2 marks]

(b) attempt to substitute 4 (seen anywhere) *(M1)*

correct equation $a=2\times 4+11$ *(A1)*

$a=19$ *A1*

[3 marks]

Total [5 marks]

6. (a) attempting to use the change of base rule *M1*

$\log_9(\cos 2x+2)=\frac{\log_3(\cos 2x+2)}{\log_3 9}$ *A1*

$=\frac{1}{2}\log_3(\cos 2x+2)$ *A1*

$=\log_3\sqrt{\cos 2x+2}$ *AG*

[3 marks]

(b) $\log_3(2\sin x)=\log_3\sqrt{\cos 2x+2}$

$2\sin x=\sqrt{\cos 2x+2}$ *M1*

$4\sin^2 x=\cos 2x+2$ (or equivalent) *A1*

use of $\cos 2x=1-2\sin^2 x$ *(M1)*

$6\sin^2 x=3$

$\sin x=(\pm)\frac{1}{\sqrt{2}}$ *A1*

$x=\frac{\pi}{4}$ *A1*

Note: Award ***A0*** if solutions other than $x=\frac{\pi}{4}$ are included.

[5 marks]

Total [8 marks]

Section B

7. (a) evidence of median position *(M1)*
80th employee
40 minutes *A1*
[2 marks]

(b) valid attempt to find interval $(25-55)$ *(M1)*
18 (employees), 142 (employees) *A1*
124 *A1*
[3 marks]

(c) recognising that there are 16 employees in the top 10% *(M1)*
144 employees travelled more than k minutes *(A1)*
$k = 56$ *A1*
[3 marks]

(d) $b = 70$ *A1*
[1 mark]

(e) (i) recognizing a is first quartile value *(M1)*
40 employees
$a = 33$ *A1*

(ii) $47 - 33$ *(M1)*
$\text{IQR} = 14$ *A1*
[4 marks]

(f) attempt to find $1.5 \times$ **their** IQR *(M1)*
$33 - 21$
12 *(A1)*
[2 marks]

[Total 15 marks]

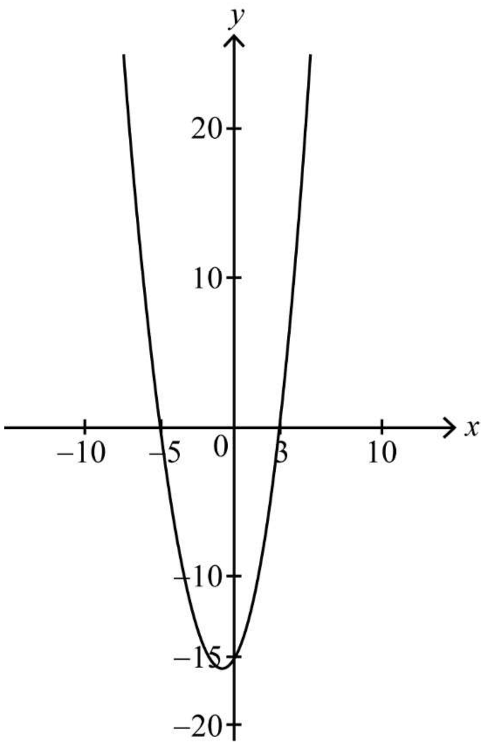
8. (a) $f'(x) = x^2 + 2x - 15$ *(M1)A1*
[2 marks]

(b) correct reasoning that $f'(x) = 0$ (seen anywhere) *(M1)*
$x^2 + 2x - 15 = 0$
valid approach to solve quadratic *M1*
$(x-3)(x+5)$, quadratic formula
correct values for x
$3, -5$
correct values for a and b
$a = -5$ and $b = 3$ *A1*
[3 marks]

continued…

Question 8 continued

(c) (i)



A1

(ii) first derivative changes from positive to negative at $x=a$ ***A1***

so local maximum at $x=a$ ***AG***

[2 marks]

(d) (i) $f''(x)=2x+2$ ***A1***

substituting **their** b into **their** second derivative ***(M1)***

$f''(3)=2\times3+2$

$f''(b)=8$ ***(A1)***

(ii) $f''(b)$ is positive so graph is concave up ***R1***

so local minimum at $x=b$ ***AG***

[4 marks]

(e) normal to f at $x=a$ is $x=-5$ (seen anywhere) ***(A1)***

attempt to find y-coordinate at their value of b ***(M1)***

$f(3)=-10$ ***(A1)***

tangent at $x=b$ has equation $y=-10$ (seen anywhere) ***A1***

intersection at $(-5,-10)$

$p=-5$ and $q=-10$ ***A1***

[5 marks]

[Total 16 marks]

9. (a) attempt to use quotient rule *(M1)*

correct substitution into quotient rule

$$f'(x)=\frac{5kx\left(\frac{1}{5x}\right)-k\ln 5x}{(kx)^2}$$ (or equivalent) ***A1***

$$=\frac{k-k\ln 5x}{k^2x^2},\left(k\in\mathbb{R}^+\right)$$ ***A1***

$$=\frac{1-\ln 5x}{kx^2}$$ ***AG***

[3 marks]

(b) $f'(x)=0$ ***M1***

$$\frac{1-\ln 5x}{kx^2}=0$$

$\ln 5x=1$ ***(A1)***

$x=\frac{\mathrm{e}}{5}$ ***A1***

[3 marks]

(c) $f''(x)=0$ ***M1***

$$\frac{2\ln 5x-3}{kx^3}=0$$

$\ln 5x=\frac{3}{2}$ ***A1***

$5x=\mathrm{e}^{\frac{3}{2}}$ ***A1***

so the point of inflexion occurs at $x=\frac{1}{5}\mathrm{e}^{\frac{3}{2}}$ ***AG***

[3 marks]

continued…

Question 9 continued

(d) attempt to integrate ***(M1)***

$u = \ln 5x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\int \frac{\ln 5x}{kx}\,\mathrm{d}x = \frac{1}{k}\int u\ \mathrm{d}u$ ***(A1)***

EITHER

$= \frac{u^2}{2k}$ ***A1***

so $\frac{1}{k}\int_1^{\frac{3}{2}} u\ \mathrm{d}u = \left[\frac{u^2}{2k}\right]_1^{\frac{3}{2}}$ ***A1***

OR

$= \frac{(\ln 5x)^2}{2k}$ ***A1***

so $\int_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \frac{\ln 5x}{kx}\,\mathrm{d}x = \left[\frac{(\ln 5x)^2}{2k}\right]_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}}$ ***A1***

THEN

$= \frac{1}{2k}\left(\frac{9}{4} - 1\right)$

$= \frac{5}{8k}$ ***A1***

setting **their** expression for area equal to 3 ***M1***

$\frac{5}{8k} = 3$

$k = \frac{5}{24}$ ***A1***

[7 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Specimen

Candidate session number

1 hour 30 minutes

Instructions to candidates

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- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

10 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

A metal sphere has a radius $12.7\,\text{cm}$.

(a) Find the volume of the sphere expressing your answer in the form $a \times 10^k$, $1 \le a < 10$ and $k \in \mathbb{Z}$. [3]

The sphere is to be melted down and remoulded into the shape of a cone with a height of $14.8\,\text{cm}$.

(b) Find the radius of the base of the cone, correct to 2 significant figures. [3]

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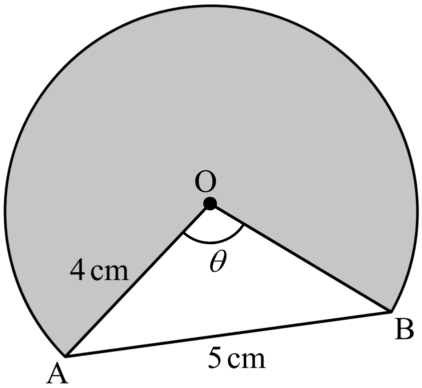
2. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius $4\,\mathrm{cm}$.



Chord AB has a length of $5\,\mathrm{cm}$ and $\mathrm{A\hat{O}B} = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

(b) Find the area of the shaded region. [3]

3. [Maximum mark: 6]

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5%, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

Laurie makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

4. [Maximum mark: 6]

A six-sided biased die is weighted in such a way that the probability of obtaining a "six" is $\frac{7}{10}$.

The die is tossed five times. Find the probability of obtaining

(a) at most three "sixes". [3]

(b) the third "six" on the fifth toss. [3]

5. [Maximum mark: 5]

The following table below shows the marks scored by seven students on two different mathematics tests.

Test 1 (x)	15	23	25	30	34	34	40
Test 2 (y)	20	26	27	32	35	37	35

Let L_1 be the regression line of x on y. The equation of the line L_1 can be written in the form $x = ay + b$.

(a) Find the value of a and the value of b. [2]

Let L_2 be the regression line of y on x. The lines L_1 and L_2 pass through the same point with coordinates (p, q).

(b) Find the value of p and the value of q. [3]

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6. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

(a) Find the maximum distance of the particle from O. [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

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Do **not** write solutions on this page.

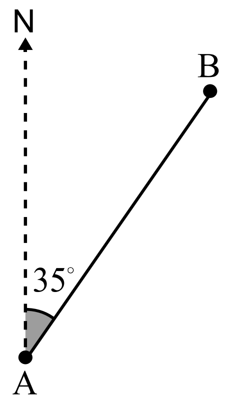
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

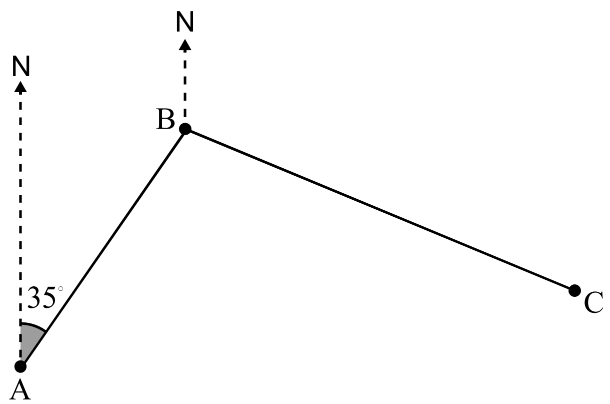
7. [Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of $4.2\,\text{km/h}$ for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a) Find the distance from point A to point B. [2]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of $4.6\,\text{km}$ until he reaches point C.



(b) (i) Show that $\text{A}\hat{\text{B}}\text{C}$ is 101°.

(ii) Find the distance from the camp to point C. [5]

(c) Find $\text{B}\hat{\text{C}}\text{A}$. [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C.

(d) Find the bearing that Jacob must take to point C. [3]

Jacob hikes at an average speed of $3.9\,\text{km/h}$.

(e) Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

The length, $X\text{ mm}$, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

(a) Find $\mathrm{P}(24.15 < X < 25)$. [2]

(b) (i) Find σ, the standard deviation of X.

(ii) Hence, find the probability that a seashell selected at random has a length greater than 26 mm. [5]

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

(c) Find $\mathrm{E}(Y)$. [3]

(d) Find the probability that exactly three of these seashells have a length greater than 26 mm. [2]

A seashell selected at random has a length less than 26 mm.

(e) Find the probability that its length is between 24.15 mm and 25 mm. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

Consider a function f, such that $f(x) = 5{,}8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

(a) Find the period of f. [2]

The function f has a local maximum at the point $(2, 21.8)$, and a local minimum at $(8, 10.2)$.

(b) (i) Find the value of b.

(ii) Hence, find the value of $f(6)$. [4]

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q$, $0 \le x \le 10$; $p, q \in \mathbb{R}$.

The function g passes through the points $(3, 2.5)$ and $(6, 15.1)$.

(c) Find the value of p and the value of q. [5]

(d) Find the value of x for which the functions have the greatest difference. [2]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

SPEC/5/MATAA/SP2/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: analysis and approaches

Standard level

Paper 2

10 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1, A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***N3***, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

Section A

1. (a) $\frac{4}{3}\pi(12.7)^3$ (or equivalent) ***A1***

8580.24 ***(A1)***

$V = 8.58\times10^3$ ***A1***

[3 marks]

(b) recognising volume of the cone is same as volume of **their** sphere ***(M1)***

$\frac{1}{3}\pi r^2(14.8) = 8580.24$ (or equivalent) ***A1***

$r = 23.529$

$r = 24\ (\text{cm})$ correct to 2 significant figures ***A1***

[3 marks]

Total [6 marks]

2. (a) ***METHOD 1***

attempt to use the cosine rule ***(M1)***

$\cos\theta = \frac{4^2+4^2-5^2}{2\times4\times4}$ (or equivalent) ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

METHOD 2

attempt to split triangle AOB into two congruent right triangles ***(M1)***

$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$ ***A1***

$\theta = 1.35$ ***A1***

[3 marks]

(b) attempt to find the area of the shaded region ***(M1)***

$\frac{1}{2}\times4\times4\times(2\pi-1.35...)$ ***A1***

$= 39.5\ (\text{cm}^2)$ ***A1***

[3 marks]

Total [6 marks]

3. (a) $\left(1+\frac{5.5}{4\times100}\right)^4$ ***(M1)(A1)***

1.056 ***A1***

[3 marks]

continued…

Question 3 continued

(b) **EITHER**

$2P = P \times \left(1 + \frac{5.5}{100 \times 4}\right)^{4n}$ **OR** $2P = P \times \left(\text{their } (a)\right)^{m}$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into loan payment formula. Award ***(A1)*** for correct substitution.

OR

$\text{PV} = \pm 1$
$\text{FV} = \mp 2$
$\text{I\%} = 5.5$
$\text{P/Y} = 4$
$\text{C/Y} = 4$
$n = 50.756\ldots$ ***(M1)(A1)***

OR

$\text{PV} = \pm 1$
$\text{FV} = \mp 2$
$\text{I\%} = 100\,(\text{their } (a) - 1)$
$\text{P/Y} = 1$
$\text{C/Y} = 1$ ***(M1)(A1)***

THEN

$\Rightarrow$ 12.7 years
Laurie will have double the amount she invested during 2032 ***A1***

[3 marks]

Total [6 marks]

4. (a) recognition of binomial ***(M1)***
$X \sim \text{B}(5, 0.7)$
attempt to find $\text{P}(X \le 3)$ ***M1***
$= 0.472 (= 0.47178)$ ***A1***

[3 marks]

(b) recognition of 2 sixes in 4 tosses ***(M1)***
$\text{P}\left(\text{3rd six on the 5th toss}\right) = \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2\right] \times 0.7 (= 0.2646 \times 0.7)$ ***A1***
$= 0.185 \left(= 0.18522\right)$ ***A1***

[3 marks]

Total [6 marks]

5. (a) $a = 1.29$ and $b = -10.4$ ***A1A1***

[2 marks]

(b) recognising both lines pass through the mean point ***(M1)***

$p = 28.7$, $q = 30.3$ ***A2***

[3 marks]

Total [5 marks]

6. (a) use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is $16.5\,\text{cm}$ (to the left of O) ***A1***

[3 marks]

(b) attempt to find time when particle changes direction *eg* considering the first maximum on the graph of s or the first t – intercept on the graph of s'. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of s' for **their** value of t, $s''(1.51986...)$ ***(M1)***

$= -8.92\ (\text{cm/s}^2)$ ***A1***

[4 marks]

Total [7 marks]

Section B

7. (a) $\frac{4.2}{60} \times 45$ ***A1***

$\text{AB} = 3.15 \text{ (km)}$ ***A1***

[2 marks]

(b) (i) 66° or $(180 - 114)$ ***A1***

$35 + 66$ ***A1***

$\text{A}\hat{\text{B}}\text{C} = 101^\circ$ ***AG***

(ii) attempt to use cosine rule ***(M1)***

$\text{AC}^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101^\circ$ (or equivalent) ***A1***

$\text{AC} = 6.05 \text{ (km)}$ ***A1***

[5 marks]

(c) valid approach to find angle BCA ***(M1)***

eg sine rule

correct substitution into sine rule ***A1***

eg $\frac{\sin(\text{B}\hat{\text{C}}\text{A})}{3.15} = \frac{\sin 101}{6.0507...}$

$\text{B}\hat{\text{C}}\text{A} = 30.7^\circ$ ***A1***

[3 marks]

(d) $\text{B}\hat{\text{A}}\text{C} = 48.267$ (seen anywhere) ***A1***

valid approach to find correct bearing ***(M1)***

eg $48.267 + 35$

bearing $= 83.3^\circ$ (accept 083°) ***A1***

[3 marks]

(e) attempt to use time $= \frac{\text{distance}}{\text{speed}}$ ***M1***

$\frac{6.0507}{3.9}$ or 0.065768 km/min ***(A1)***

$t = 93$ (minutes) ***A1***

[3 marks]

Total [16 marks]

8. (a) attempt to use the symmetry of the normal curve *(M1)*

eg diagram, $0.5-0.1446$

$\mathrm{P}(24.15 < X < 25) = 0.3554$ ***A1***

[2 marks]

(b) (i) use of inverse normal to find z score *(M1)*

$z = -1.0598$

correct substitution $\frac{24.15-25}{\sigma} = -1.0598$ *(A1)*

$\sigma = 0.802$ ***A1***

(ii) $\mathrm{P}(X > 26) = 0.106$ *(M1)**A1***

[5 marks]

(c) recognizing binomial probability *(M1)*

$\mathrm{E}(Y) = 10 \times 0.10621$ *(A1)*

$= 1.06$ ***A1***

[3 marks]

(d) $\mathrm{P}(Y = 3)$ *(M1)*

$= 0.0655$ ***A1***

[2 marks]

(e) recognizing conditional probability *(M1)*

correct substitution ***A1***

$\frac{0.3554}{1-0.10621}$

$= 0.398$ ***A1***

[3 marks]

Total [15 marks]

9. (a) correct approach ***A1***

eg $\frac{\pi}{6} = \frac{2\pi}{period}$ (or equivalent)

$\text{period} = 12$ ***A1***

[2 marks]

(b) (i) valid approach *(M1)*

eg $\frac{\max + \min}{2}$ $b = \max - \text{amplitude}$

$\frac{21.8+10.2}{2}$, or equivalent

$b = 16$ ***A1***

continued…

Question 9 continued

(ii) attempt to substitute into **their** function ***(M1)***

$5.8\sin\left(\frac{\pi}{6}(6+1)\right)+16$

$f(6)=13.1$ ***A1***

[4 marks]

(c) valid attempt to set up a system of equations ***(M1)***
two correct equations ***A1***

$p\sin\left(\frac{2\pi}{9}(3-3.75)\right)+q=2.5$, $p\sin\left(\frac{2\pi}{9}(6-3.75)\right)+q=15.1$

valid attempt to solve system ***(M1)***
$p=8.4$; $q=6.7$ ***A1A1***

[5 marks]

(d) attempt to use $|f(x)-g(x)|$ to find maximum difference ***(M1)***

$x=1.64$ ***A1***

[2 marks]

Total [13 marks]