IB/ A-level differentiation

Addvance
AddvanceMaths.com/Guides

Equation of a tangent

Instructions:
To calculate the equation of a tangent to a curve at a specific point, follow the steps below:

Calculate the y-coordinate. → **Find the derivative.** → **Calculate the gradient.** → **Substitute to find c.**

Example: Calculate the equation of the tangent of the function $f(x) = 4x^2 + 3x + 7$ when $x = 2$.

Step one: Calculate the y-coordinate by substituting the x value into the function.

$x=2$
$y=4(2)^2+3(2)+7$
$y=16+6+7 = 29$

Step two: Find the derivative (using a rule, such as the power rule).

$f(x)=4x^2+3x+7$
$f'(x)=8x+3$

Step three: Substitute the x value into the derivative to calculate the gradient of the function at the specified point.

$x=2$
$f'(2)=8(2)+3$
$f'(2)=16+3 = 19$

Step four: Substitute the x and y co-ordinates and the gradient into the equation $y = mx + c$ to calculate c.

$y=mx+c$
$29=19(2)+c$
$29=38+c$
$c=29-38= -9$

$y=19x-9$ ✓

1. Calculate the equation of the tangent to the curve $y = 3x^2 + 4$ at the point $(4{,}52)$.

$y =$

(3)

---

2. At the point $(1{,}24)$, calculate the equation of the tangent to the curve $y = 4x^3 + 9x^2 + 45x + 8$.

$y =$

(3)

3. Calculate the equation of the tangent to the curve $y = \frac{2}{x^2}$ at the point $(1,2)$.

(4)

4. Calculate the equation of the tangent to the curve $y = 2x(x-5)^2$ when $x = 7$.

$y =$

(4)

5. At the minimum, calculate the equation of the tangent to the curve $y = 4x^3 + 8x^2 + 5x + 4$.

$y =$

(4)

---

6. At the maximum, calculate the equation of the tangent to the curve $y = 4x(3x + 3)^2$**.**

$y =$

(4)

7. At the point of inflection, calculate the equation of the tangent to the curve $y = 2x^3 + 3x^2 + 2x + 1$.

In the form $ax + by = c$.

(5)

---

8. Calculate the equation of the tangent to the curve $y = \frac{6}{x^4}$ at the where the gradient of the line, $m = 0.75$.

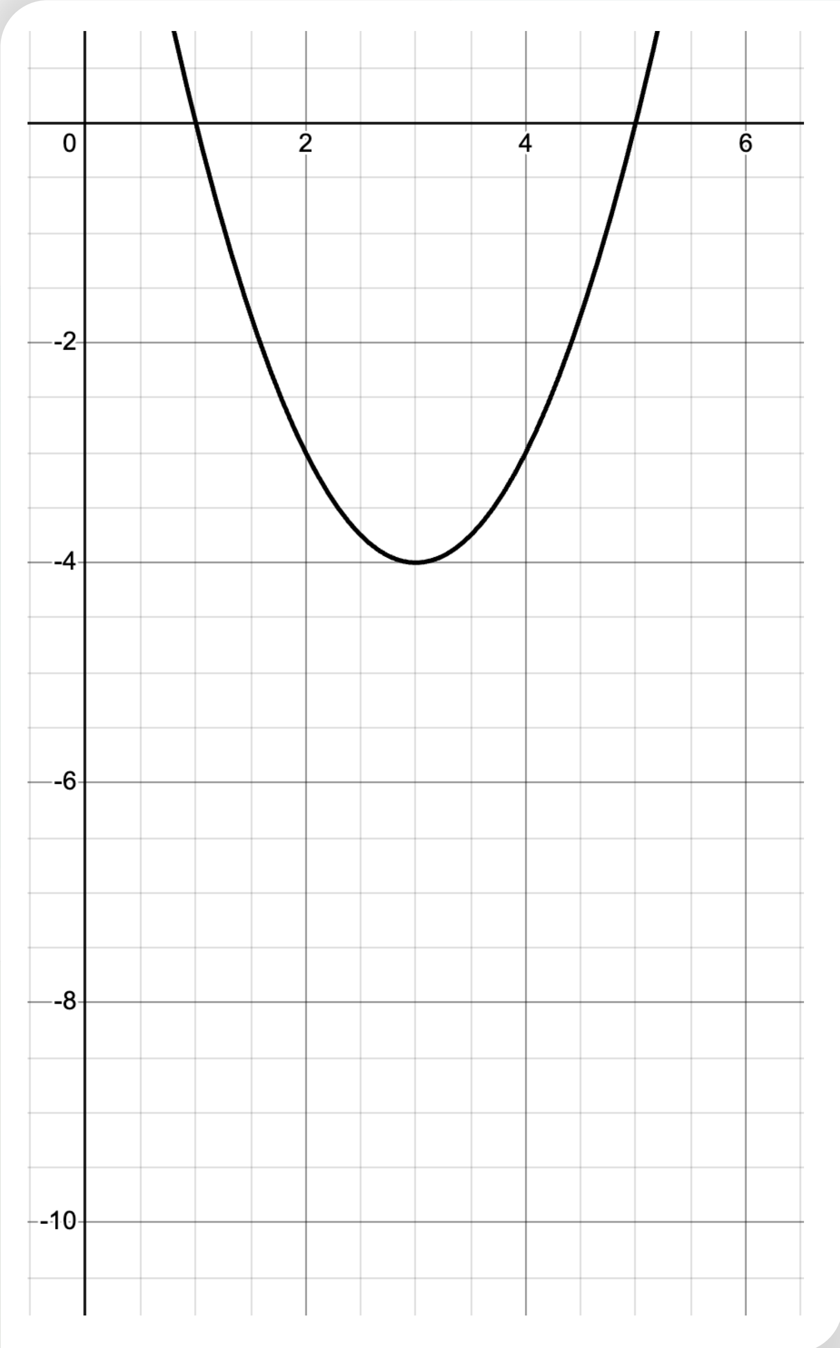
In the form $ax + by = c$.

(5)

IB/ A-level differentiation

Equation of a tangent

9. The graph below shows the quadratic function $f(x)$. Construct the tangent to the curve when $x = 4$.



a. Identify the function

Hint: Find the roots of the graph to write the function in the form $y = (x - a)(x - b)$.

(2)

b. Find the equation of the tangent when $x = 4$.

$y =$

(3)

c. Construct the tangent accurately on the graph.

(1)

10. The function $f(x) = 2x^2 + 3x + 5$.

a. Calculate the equation of the tangent at point $(a, 10)$ **where** $a > 0$.

(5)

b. Calculate the equation of the normal at the same point.

Hint: The normal is perpendicular to the tangent and passes through the intersection.

In the form $ax + by = c$.

(3)

11. The function $f(x) = x^3 - 4x^2 + 4x - 4.$

a. Calculate the equation of the tangent at the point of inflection.

(5)

b. Calculate the equation of the normal at the minimum.

(5)

12. The function $f(x) = -2x^2 + 4x + 4$

a. Calculate the equation of the tangent at the point $(b, 4)$**, for** $b > 0$**.**

(4)

b. Calculate the equation of the normal at the same point.

In the form $ax + by = c$.

(4)

CHALLENGE

13. The equation of the tangent to the curve $f(x) = \frac{a}{\sqrt{x}}$ is $y = -\frac{1}{a}x + 3$ when $y = 2$. Calculate the equation of the normal to the curve at the point $(1,4)$.

Hint: Start by substituting the given value of y into the equation of the tangent.

In the form $ax + by = c$ ***where*** $a > 0$.

(6)