Algebraic Proofs

GCSE Worksheet

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1. Prove that the sum of two odd numbers is always even.

(3)

2. Prove that the difference of the squares of any two integers is always divisible by their sum and difference.

(2)

3. Prove that the square of an odd number is always odd.

(3)

4. Prove that $n^3 - n$ is a multiple of 6 for any integer n.

5. Prove that that the square of any 2-digit integer ending in 5 ends in 25. (3)

(3)

6. Prove that the difference between the squares of two consecutive integers is always odd.

(3)

7. Prove that the sum of four consecutive integers is always even.

8. Prove that the cube of an odd number is always odd. (2)

(4)

9. Prove that the product of two consecutive even numbers is divisble by 8.

(3)

10. Prove algebraically that the difference between the squares of two consecutive even integers is divisible by 4.

(3)

11. Prove that the integer length of the hypotenuse of a right angled triangle is an even number if the two other side lengths are even.

(4)

12. a. Show that $4^{50} - 1$ is the product of two consecutive odd numbers.

(2)

b. Ahmed says, "$3^{50} - 1$ is also a product of two consecutive odd numbers."

Is Ahmed correct? You must give a reason for your answer.

(1)

13. Prove algrebraically that $(n-1)^3 + (n+1)^3 - 2n^3$ is divisible by 6 for all integers n.

(4)

14. Prove that $(n+1)^4 - (n-1)^4$ is divisble by 8 for all integers n.

15. Prove that the sum of four odd integers is divisible by 2. (4)

16. Prove that $n + n^2$ is always even for all integers n. (2)

(2)

17. **CHALLENGE!** Prove that $n^5 - 5n^3 + 4n$ is the product of five consecutive integers.

(4)

18. **MEGA-CHALLENGE!** Prove that the product of four consecutive integers plus 1 is a perfect square.

(5)