# Bounds

*MARK SCHEME*

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**1.** State the upper and lower bounds of:

a) 150cm measured to the nearest 10 cm

Upper: 155

Lower: 145 (2)

b) 250g measured to the nearest 10g

Upper: 255

Lower: 245 (2)

c) 3.6m measured to the nearest 0.1m

Upper: 3.65

Lower: 3.55 (2)

d) 7.8Kg measured to the nearest 0.1kg.

Upper: 7.85

Lower: 7.75 (2)

e) 1200 ml measured to the nearest 100ml.

Upper: 1250

Lower: 1150 (2)

**2.** John's room is 10m wide, rounded to one significant figure. What are the upper and lower bounds?

**Note:** While it may appear that 5 is the lower bound, it is not. If x were 5, that is already 1 significant figure. If x were for example, 9.4, that rounds to 9, not 10.

Upper: 15

Lower: 9.5 (2)

**3.** The following values of $x$ have been rounded to the nearest 100. Express the range of values of $x$ before rounding as an inequality.

a) 300

$250 \leq x < 350$ ..... (2)

b) 1200

$1150 \leq x < 1250$ ..... (2)

c) 4500

$4450 \leq x < 4550$ ..... (2)

d) 9800

$9750 \leq x < 9850$ ..... (2)

e) 76500

$76450 \leq x < 76550$ ..... (2)

**Note:** 1 mark given for correct values, and 1 mark given for correct inequality signs.

**4.** A square has side lengths of 5cm, rounded to the nearest whole number. Calculate the upper bound of the area of the square.

$4.5 \leq l < 5.5$ **M1** for calculating upper bound

$(5.5)^2 = 30.25$ **M2** for squaring upper bound

$30.25$ cm ..... (2)

**5.** A triangle has a height of 50cm and base of 90cm, both rounded to the nearest 10cm. Calculate the lower bound for the area of the triangle.

$45 \leq h < 55$
$85 \leq b < 95$ **M1** for calculating lower bounds

$a = \frac{1}{2}bh$ **M2** for correct formula **OR** substitution

$a = \frac{1}{2}(45)(85) = 1912.5$ **M3** for correct answer

$1912.5$ cm² ..... (3)

**6.** 20 cubes each have a side length of 15cm, rounded to the nearest whole number. Calculate the minimum volume of a box needed to pack all the cubes.

$14.5 \leq s < 15.5$

$(14.5)^3 = 3048.625$

$3{,}048.625 * 20 = 60{,}972.5$

**M1** for calculating lower bound

**M2** for cubing lower bound

**M3** for multiplying by 20

$60{,}972.5cm^3$

..... (3)

**7.** Will has three sofas of lengths 209cm, 187cm and 132cm, rounded to the nearest centimeter. He needs to place these along a wall of 526.2cm. Can this even be possible? Explain your answer.

$208.5 \leq l_1 < 209.5$

$186.5 \leq l_2 < 187.5$

$131.5 \leq l_3 < 132.5$

**M1** for calculating lower bounds

$208.5 + 186.5 + 131.5 = 526.5cm$

**M2** for adding values

The sum of the lower bounds of the three lengths is 526.5 cm, which is greater than the wall length, 526.2cm.

**M3** for appropriate justification

(3)

**8.** A car travels 40km in 0.3h, rounded to the nearest 0.1h. Calculate the lower bound of the average speed. Give your answer to the 3 significant figures.

$0.25 \leq t < 0.35$

**M1** for calculating **upper bound** of time

$s = \frac{D}{t}$

$s = \frac{40}{(0.35)}$

**M2** for correct formula **OR** substitution

$s = 114.2857142857 \ldots$

**M3** for correct answer **AND** rounding

$114\ km/h$

..... (3)

**9.** A marble has a density of 3g/cm$^3$, rounded to the nearest whole number.

Lua has a bag of 20 marbles, each of which has a volume of 2cm$^3$, rounded to the nearest whole number.

The bag weighs 5.6g, rounded to 1 decimal place. Calculate the maximum mass of the bag.

$2.5 \leq \rho < 3.5$

$1.5 \leq v < 2.5$

$5.55 \leq b < 5.65$

**M1** for correct bounds

$20 \times 2.5 = 50$

**M2** for multiplication by 20

$m = \rho v = 3.5 * 50 = 175$

**M3** for calculating volume

$175 + 5.65 = 180.65$

**M4** for correct answer

$180.65g$ ..... (4)

**10.** The acceleration of a car is given by:

$$a = \frac{v - u}{t}$$

$v = 9.5$m/s, rounded to 1 decimal place
$u = 7.84$m/s, rounded to 2 decimal places
$t = 3$s, rounded to the nearest whole number

Calculate the upper bound of the car's acceleration. Give your answer to 1d.p.

$9.45 \leq v < 9.55$

$7.835 \leq u < 7.845$

$2.5 \leq t < 3.5$

**M1** for correct bounds

$a = \frac{v_{upper} - u_{lower}}{t_{lower}}$

**M2*** for identifying upper/lower or max/min for v, u, t
*Award M2 mark for correct M3 if M2 missing

$a = \frac{9.55 - 7.835}{2.5}$

**M3** for correct substitution

$a = 0.686$

**M4** for correct answer **AND** rounding

$0.7$ ..... m/s$^2$ (4)

**11.** The displacement of a plane is given by: $s = ut + \frac{1}{2}at^2$

$u = 3.5$m/s, rounded to 1 decimal place
$t = 4$s, rounded to the nearest whole number
$a = 9.81$m/s², rounded to 2 decimal places

Calculate the lower bound of the plane's displacement. Give your answer to 2.d.p.

$3.45 \leq u < 3.55$

$3.5 \leq t < 4.5$ **M1** for correct bounds

$9.805 \leq a < 9.815$

$s = (3.45)(3.5) + \frac{1}{2}(9.805)(3.5)^2$ **M2** for correct substitution

$s = 72.130625$ **M3** for correct answer **AND** rounding

72.13 m (3)

**12.** $$k = \frac{2v - 3m^2}{\sqrt{(2s)}}$$

$v = 6.9$, rounded to 1 decimal place
$m = 70$, rounded to the nearest 10
$s = 150$, rounded to 2 significant figures

Calculate the upper bound of $k$. Give your answer to 3 significant figures.

$6.85 \leq v < 6.95$

$65 \leq m < 75$ **M1** for correct bounds

$145 \leq s < 155$

$k = \frac{2(v_{upper}) - 3(m_{lower})^2}{\sqrt{(2(s_{lower}))}}$ **M2*** for identifying upper/lower or max/min for v, m, s
*Award M2 mark for correct M3 if M2 missing

$k = \frac{2(6.95) - 3(65)^2}{\sqrt{(2(145))}}$ **M3** for correct substitution

$k = -743.4853921$ **M4** for correct answer **AND** rounding $-743$ (4)

**13.** $$n = \frac{m^2 + 4v}{3k}$$

$m = 4.5$, rounded to 1 decimal place
$v = 20.1$, rounded to 1 decimal place
$k = 7.54$, rounded to 2 decimal places

By considering bounds, work out the value of $n$ to a suitable degree of accuracy.
Give a reason for your answer.

$4.45 \leq m < 4.55$

$20.05 \leq v < 20.15$

$7.535 \leq k < 7.545$

**M1** for 4/5 correct bounds

**M2** for all 6 correct bounds

Upper bound

$$n = \frac{(m_{upper})^2 + 4(v_{upper})}{3(k_{lower})}$$

$$n = \frac{(4.55)^2 + 4(20.15)}{3(7.535)}$$

**M3** for correct upper bound substitution

$n = 4.48142004 \ldots$

Lower bound

$$n = \frac{(m_{lower})^2 + 4(v_{lower})}{3(k_{upper})}$$

$$n = \frac{(4.45)^2 + 4(20.05)}{3(7.545)}$$

**M4** for correct lower bound substitution

$n = 4.418047272 \ldots$

$n =$ 4

Because both the upper and lower bound round to 4 **OR** to 1 s.f. **OR** to nearest whole number

**M5** for correct answer **AND** reason

(5)