Year 11 Topics 1-9 exam

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Questions and solutions by Imad Raheel

1. Rearrange the following to make x the subject of the formula, showing all your working.

(a) $x^2 + 11x + 11 = 7 + 5x + y$

$x^2 + 6x + 4 = y$

$y = (x+3)^2 - 9 + 4$

$y = (x+3)^2 - 5$

$y + 5 = (x+3)^2$

$\pm\sqrt{y+5} = x+3 \Rightarrow x = -3 \pm \sqrt{y+5}$

x = $-3 \pm \sqrt{y+5}$ 3

(b) ~~$ax^2 + bx + c = 0$~~ $2y = \frac{2x+y}{x+3}$

$2y(x+3) = 2x + y$

$2xy + 6y - 2x = y$

$x(2y-2) = -5y$

$x = \frac{-5y}{2y-2}$

x = $\frac{-5y}{2y-2}$ 4

(c) ~~Hence, or otherwise~~, solve $2x^2 + \frac{6x+3}{2} = 1$

$\frac{4x^2+6x+3}{2} = 1$

$4x^2 + 6x + 3 = 2$

$4x^2 + 6x + 1 = 0$

Using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$,

$x = \frac{-3 \pm \sqrt{5}}{4}$

x = $\frac{-3 \pm \sqrt{5}}{4}$ 3

2. A $1 coin weighs 8.75 g, correct to the nearest 0.01 g.
Adarsh weighs the contents of a large bag of $1 coins.
The coins weigh 2.63 kg, correct to the nearest 10 g.
What's the maximum amount ~~Adarsh will have to pay~~? of coins Adarsh has?

Upper bound required

$8.745 \leq w < 8.755$

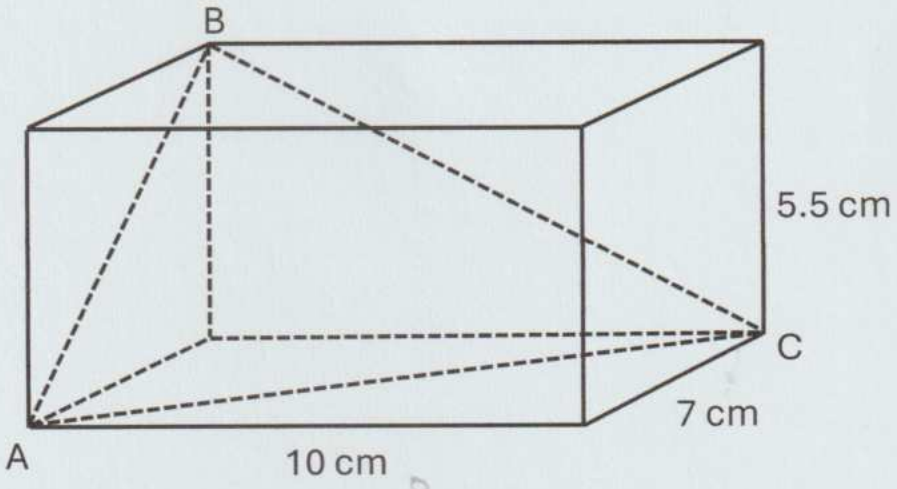
$2625\text{ g} \leq W_b < 2635\text{ g}$

$$\text{Coins}_{max} = \frac{W_{b\,max}}{w_{min}} = \frac{2635}{8.745} = 301.3\ldots$$

So, max no. of coins is 301

301

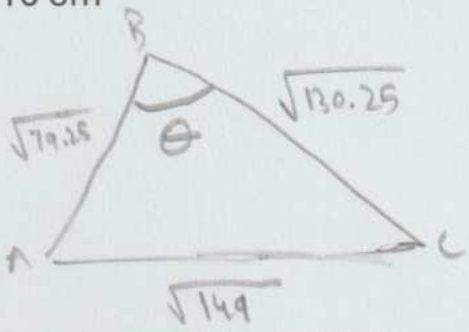
3. The figure shows a cuboid with dimensions 10 cm x 7 cm x 5.5 cm.
The cuboid is cut by Kyran so that the knife passes through the points A, B and C.
Both the blocks are to be fully painted.
A tub of paint containing 75 cm^3 of paint costs \$4.49.
How much will Kyran have to pay?



$AC = \sqrt{10^2+7^2} = \sqrt{149}$

$AB = \sqrt{7^2+5.5^2} = \sqrt{79.25}$

$BC = \sqrt{10^2+5.5^2} = \sqrt{130.25}$



$\sqrt{149}^2 = \sqrt{79.25}^2 + \sqrt{130.25}^2 - 2\times\sqrt{79.25}\times\sqrt{130.25}\times\cos\theta$

$149 = 79.25+130.25 - 2\times\sqrt{79.25}\times\sqrt{130.25}\times\cos\theta$

$-60.5 = -2\sqrt{79.25}\sqrt{130.25}\cos\theta$

$\cos\theta = \frac{60.5}{2\sqrt{79.25}\sqrt{130.25}}$

$\theta = \cos^{-1}(0.298)$

$\theta = 72.7$

$A_{ABC} = \frac{1}{2}\times\sqrt{79.25}\times\sqrt{130.25}\times\sin 72.7$

$= 48.5$

$S.A_{paint} = 2(10\times7 + 10\times5.5 + 7\times5.5) + 2\times48.5$

$= 423.99\ cm^2$

S.A paint $= 423.99$

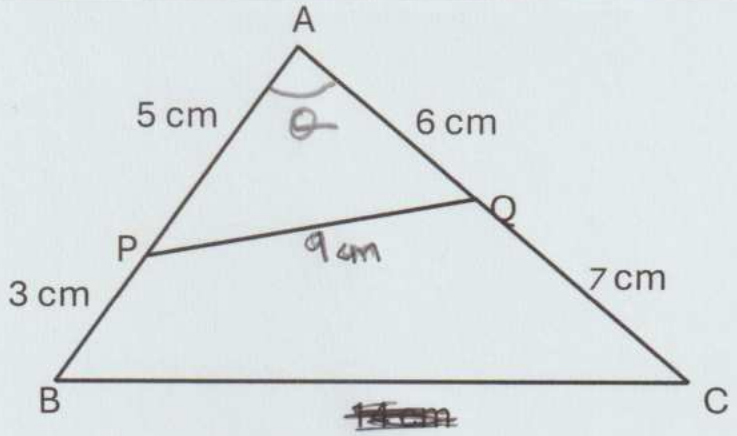
Tubs needed $= \frac{423.99}{75} = 6$

Money $= 6\times4.49$

$= \$26.94$

\$26.94

4. In the figure, the point P lies on AB such that AP = 5 cm and PB = 3 cm. The point Q lies on AC such that AQ = 6 cm and QC = 7 cm. Given that PQ is 9 cm, find the length of BC.



$9^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos\theta$

$9^2 - 5^2 - 6^2 = -60\cos\theta$

$20 = -60\cos\theta$

$-\frac{1}{3} = \cos\theta$

$\theta = \cos^{-1}(-1/3)$

$\theta = 109.5^\circ$

Let BC = x

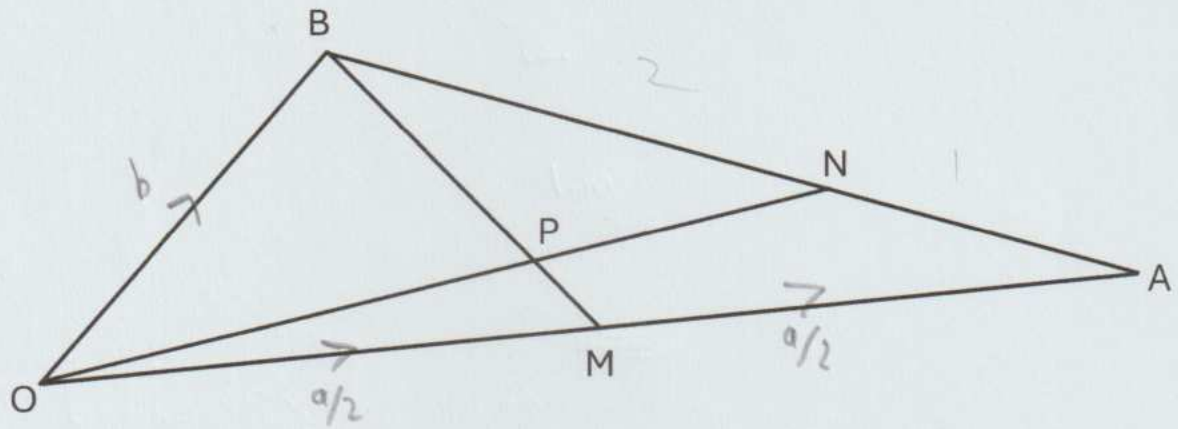
$x^2 = 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos(109.5)$

$x^2 = \frac{907}{3}$

$x = \sqrt{\frac{907}{3}} = 17.4$

17.4

5. In the diagram, *OAB* is triangle with $\overrightarrow{OA}$ = a, and $\overrightarrow{OB}$ = b.
M is the midpoint of OA. N is a point on AB such that AN:AB = 1:3 and P is the point of intersection of ON and BM.
Find the ratio of $\overrightarrow{OP}$: $\overrightarrow{PN}$, giving your answer in the form 1:n .



$\overrightarrow{AB} = b - a$

$\overrightarrow{AN} = \frac{b-a}{3}$

$\overrightarrow{ON} = a + \frac{b-a}{3} = \frac{3a+b-a}{3} = \frac{2a+b}{3}$

$\overrightarrow{OP} = x\left(\frac{2a+b}{3}\right) = \left(\frac{2x}{3}\right)a + \left(\frac{x}{3}\right)b$ — ①

$\overrightarrow{MB} = b - \frac{a}{2} = \frac{2b-a}{2}$

$\overrightarrow{MP} = y\left(\frac{2b-a}{2}\right)$

$\overrightarrow{OP} = \frac{a}{2} + \frac{2yb - ya}{2}$

$= \frac{(1-y)a + (2y)b}{2}$

$= \left(\frac{1-y}{2}\right)a + yb$ — ②

Using eq ① and ②,

$\frac{2x}{3} = \frac{1-y}{2}$ $\quad \frac{x}{3} = y$

$4x = 3 - 3y$

$3y = 3 - 4x$

$\frac{3x}{3} = 3 - 4x$

$5x = 3$

$x = 3/5$

$\overrightarrow{OP} = x(\overrightarrow{ON})$

$\overrightarrow{OP} = \frac{3}{5}(\overrightarrow{ON})$

$\frac{\overrightarrow{OP}}{\overrightarrow{ON}} = \frac{3}{5}$

$\overrightarrow{OP} : \overrightarrow{ON} = 3:5$

$\therefore \overrightarrow{OP} : \overrightarrow{PN} = 3:2$

$= 1 : 2/3$

1 : $\frac{2}{3}$

6. The functions f and g are such that;

$$f(x) = 2x + 1 \qquad g(x) = x^2 - 1$$

(a) Find all values of x for which $gf(x) = g(x)$

$gf(x) = (2x+1)^2 - 1 = 4x^2 + 4x + 1 - 1$

$4x^2 + 4x = x^2 - 1$

$3x^2 + 4x + 1 = 0$

$3x^2 + 3x + x + 1 = 0$

$3x(x+1) + 1(x+1) = 0$

$(x+1)(3x+1) = 0$

$x = -1$ or $x = -1/3$

x = -1 or -1/3 3

(b) Find $gf^{-1}(x)$

$gf(x) = 4x^2 + 4x$

$y = 4x^2 + 4x$

$x = 4y^2 + 4y$

$x = 4(y^2 + y)$

$x = 4\left[(y + 1/2)^2 - \frac{1}{4}\right]$

$x = 4(y + \frac{1}{2})^2 - 1$

$\frac{x+1}{4} = (y + \frac{1}{2})^2$

$y + \frac{1}{2} = \pm\sqrt{\frac{x+1}{4}}$

$y = \frac{-1}{2} \pm \frac{\sqrt{x+1}}{2}$

$y = \frac{-1 \pm \sqrt{x+1}}{2}$

$gf^{-1}(x) = \frac{-1 \pm \sqrt{x+1}}{2}$

$gf^{-1}(x) = \frac{-1 \pm \sqrt{x+1}}{2}$ 5

7. Solve the following, showing all your working

(a) $2x^2 + 7x + 3 = 0$

x = $-1/2$ or -3 3

(b) $3x^2 - 10x + 7 = 0$

x = $7/3$ or 1 3

(c) $4x^2 - 16x = -15$

$4x^2 - 16x + 15 = 0$

x = $5/2$ or $3/2$ 3

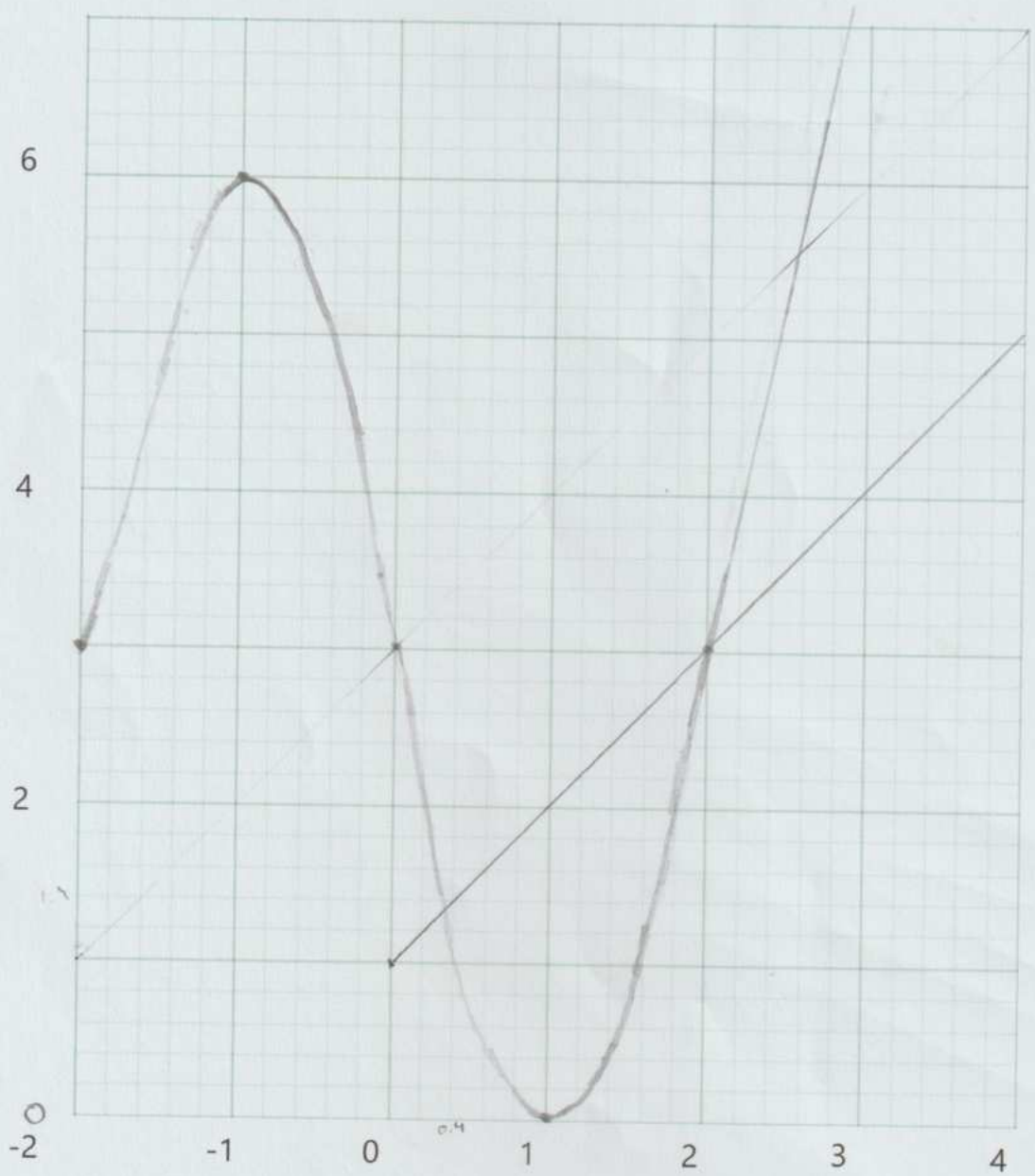
8. A graph has equation $y = x^3 - 4x + 3$

(a) Fill in the table with the missing values of y

x	-2	-1	0	1	2	3	4
y	3	6	3	0	3	18	51

3

(b) Plot the graph $y = x^3 - 4x + 3$



4

(c) Find the coordinates of the turning point(s) of the graph, showing all your working

$y = x^3 - 4x + 3$

$\frac{dy}{dx} = 3x^2 - 4$

$3x^2 - 4 = 0$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \frac{2}{\sqrt{3}}$ $\quad x = \frac{-2}{\sqrt{3}}$

$y = -0.08$ $\quad y = 6.08$

$(2/\sqrt{3}, -0.08)$ or $(-2/\sqrt{3}, 6.08)$

..... 4

(d) Use your graph to find estimates for solutions to $x^3 - 5x + 3 = 1$
No credit is awarded for solving the equation

$x^3 - 4x + 3 - x = 1$

$x^3 - 4x + 3 = x + 1$

$x = 0.4$

$x = 2$

..... 3

(e) Find the gradient of the graph at the point where the graph intersects the line x = 4

(4, 51)

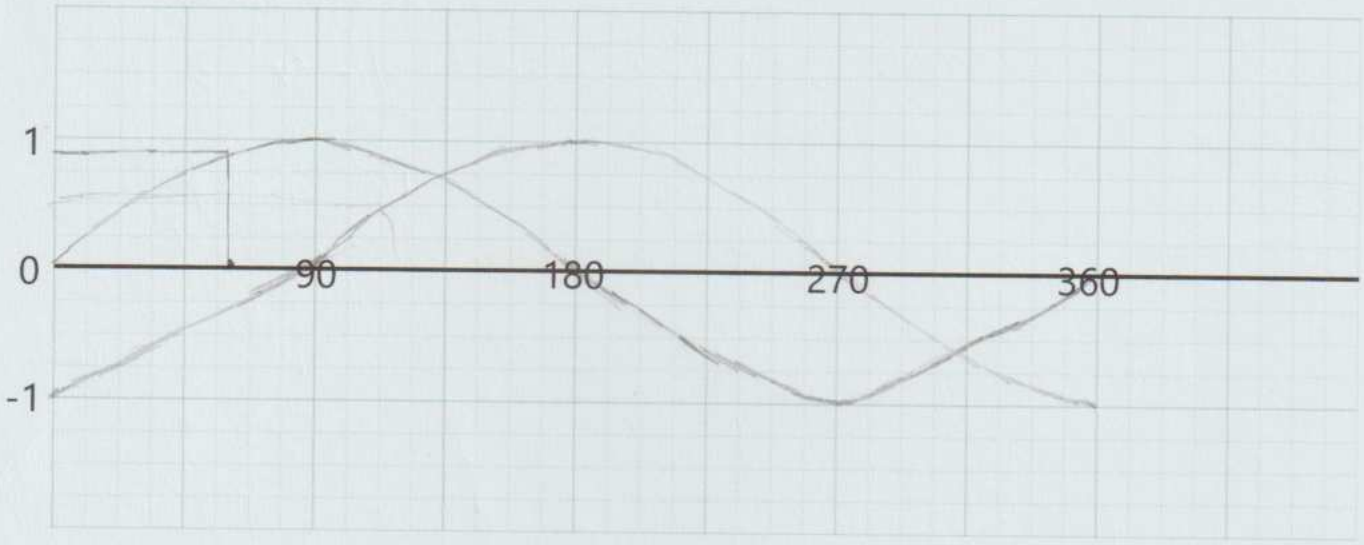
$\frac{dy}{dx} = 3x^2 - 4$

$3 \times 4^2 - 4 = 44$

44

..... 3

9. (a) Sketch the graph of y = sin(x)



(b) Hence, sketch the graph of y = sin(x-90)

(c) Use your graph to estimate solutions to y=sin(60)

(60.....) , (0.8.....) 2

(d) Use your graph to estimate solutions to sin(x-90) = 0.5

(135.....) , (0.5.....) 3

10. The 3rd term of an arithmetic series is 42, and the 10th term is 0.
Given that $S_n = 0$, find the value of n.

$D = \frac{42-0}{10-3} = 42/7 = 6$

1st + 1st term $= 42 + 12 = 54$

$\frac{n}{2}[2 \times 54 + 6(n-1) = 0]$

$108 + 6n - 6 = 0$

$102 = -6n$

$n = 17$

n = 17 5

END OF EXAM

Total marks available: 80

Remember to re-check your answers