GCSE Maths

Quadratic Inequalities

Scaffolded Practice

www.addvancemaths.com/revision/quadratic-inequalities/

Video Lesson and Mark Scheme

Name:	
Score:	
Time:	

Instructions

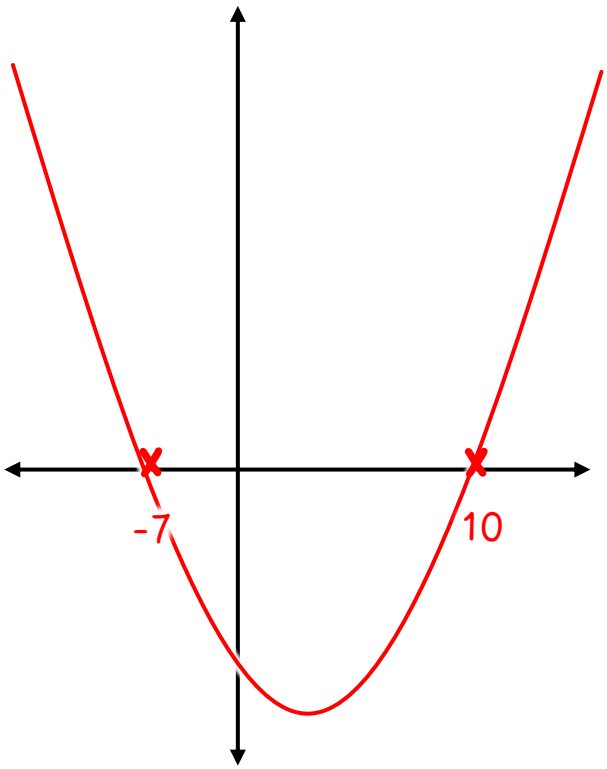
- Use black or blue ball-point pen.
- Answer all the questions in the spaces provided.

Information

- The marks for each question are shown in the circles.
- You are allowed a calculator for this test.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.



Quadratic Inequalities

Let's practice quadratics…

Factorise:

$x^2 - 6x + 8 = (x-4)(x-2)$

$x^2 - 25x + 24 = (x-1)(x+24)$

$x^2 - 25 = (x-5)(x+5)$

$x^2 - 9x = x(x-9)$

$x^2 - x - 56 = (x-8)(x+7)$

$x^2 - 10x + 16 = (x-8)(x-2)$

5

Solve these equations by factorising:

$x^2 - 6x + 8 = 0$
$(x-4)(x-2) = 0$
$x = 4, x = 2$

$x^2 + 4x + 3 = 0$
$(x+3)(x+1) = 0$
$x = -3 \quad x = -1$

$x^2 - 16 = 0$
$(x-4)(x+4) = 0$
$x = 4 \quad x = -4$

$x^2 - 6x = 0$
$x(x-6)$
$x = 0 \quad x = 6$

$x^2 - 14x + 45 = 0$
$(x-9)(x-5) = 0$
$x = 9 \quad x = 5$

$x^2 + 2x - 80 = 0$
$(x-8)(x+10) = 0$
$x = 8, \quad x = -10$

$x^2 + 30 = 13x$
$x^2 - 13x + 30 = 0$
$(x-10)(x-3) = 0$
$x = 10 \quad x = 3$

$x^2 + x = x + 36$
$x^2 - 36 = 0$
$(x-6)(x+6) = 0$
$x = 6 \quad x = -6$

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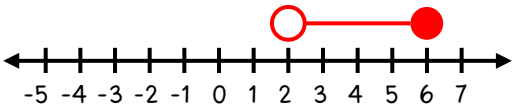
Quadratic Inequalities

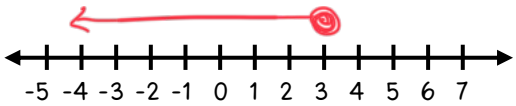
Let's practice inequalities…

On the lines below draw the inequalities.
The first one has been done for you.

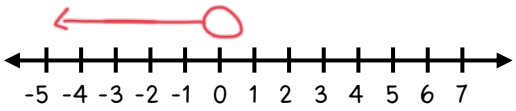
$2 < x \leq 6$



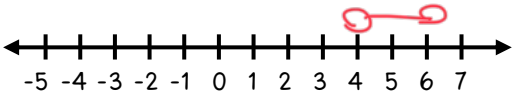
$x \leq 3$



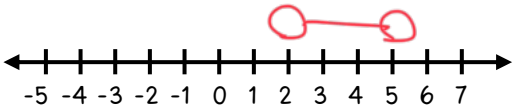
$x < 0$



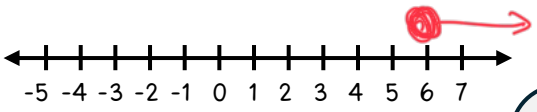
$4 < x < 6$



$2 < x < 5$



$x \geq 6$



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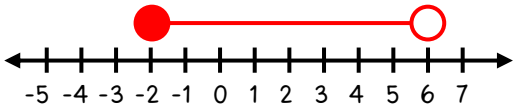
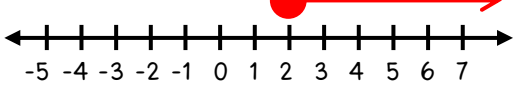
Write down the inequality represented on the number line.
The first one has been done for you.

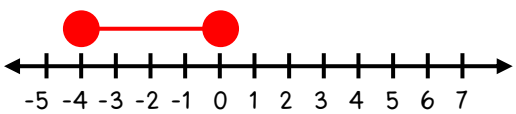
$-2 \leq x < 6$



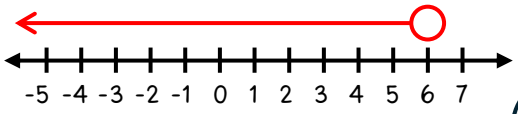
$x \geq 2$



$-4 \leq x \leq 0$



$x < 6$



3

List the integers that satisfy these two inequalities:

a) $2 \leq x < 7$ 2, 3, 4, 5, 6

b) $-3 < x < 3.5$ −2, −1, 0, 1, 2, 3

2

Quadratic Inequalities

Let's practice sketching quadratic graphs.

On the axes below, complete the sketch for the quadratic graph.
You must label the roots (x intercepts) on your sketch.
An example has been done for you.

Watch this video for help!

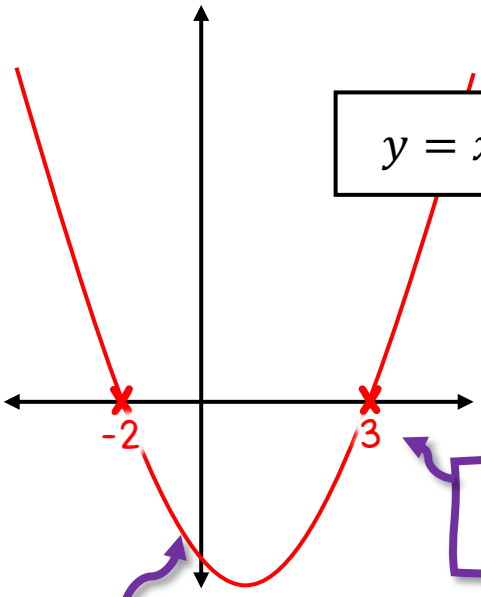
$$y = x^2 - x - 6$$

Step 1: Make the equation equal to zero and factorise the quadratic

$(x-3)(x+2) = 0$

Step 2: Solve to find the roots.

$x = 3$, $x = -2$

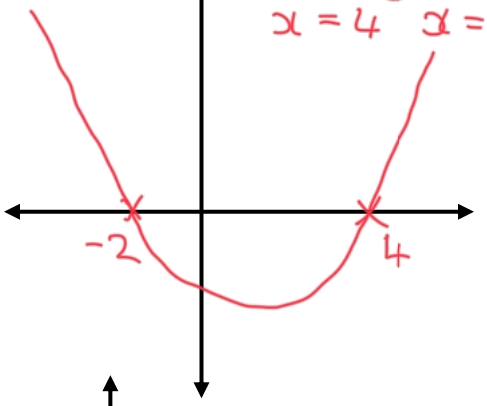


Step 3: Put crosses where the roots should go on the x- axis.

Step 4: Draw a u – shaped curve passing through these roots.

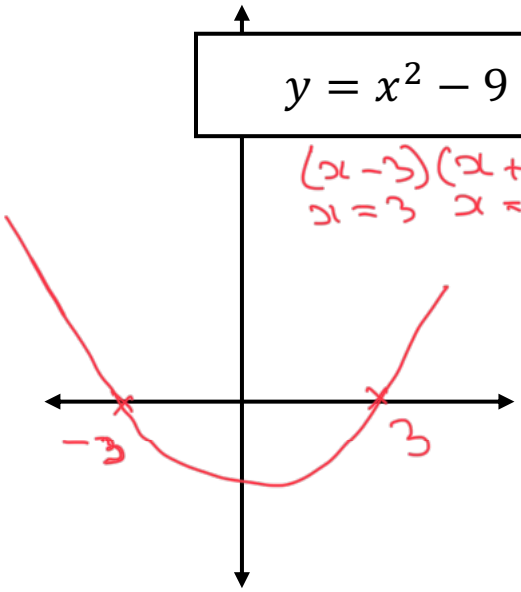
$$y = x^2 - 2x - 8$$

$(x-4)(x+2)$
$x = 4$ $x = -2$



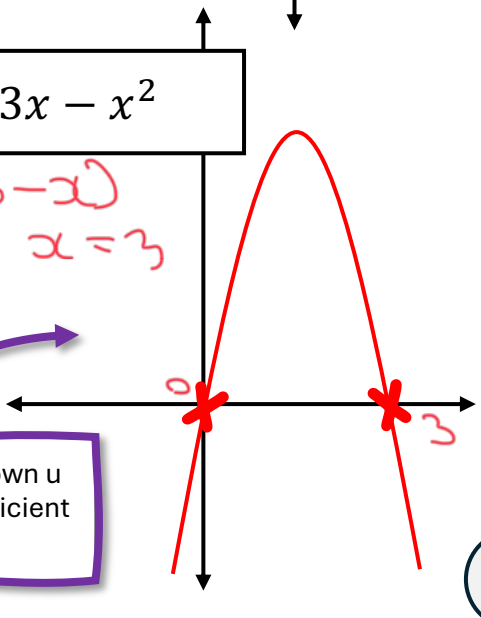
$$y = x^2 - 9$$

$(x-3)(x+3)$
$x = 3$ $x = -3$



$$y = 3x - x^2$$

$x(3-x)$
$x = 0$ $x = 3$



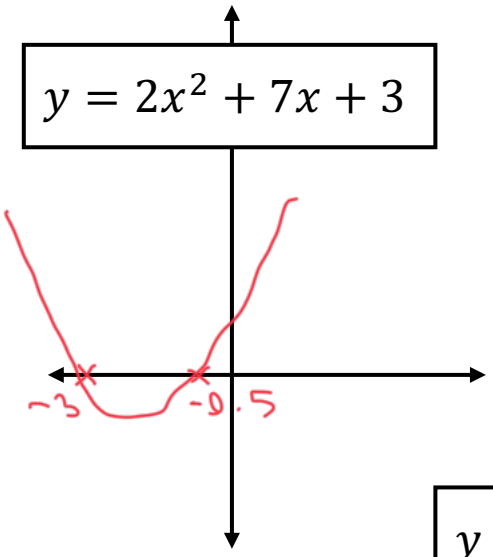
This one is an upside-down u shape because the coefficient of x^2 is negative.

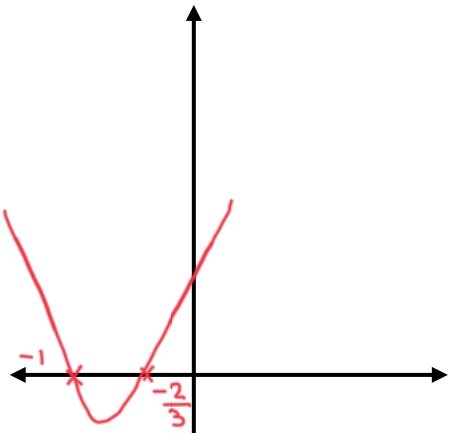
Quadratic Inequalities

Let's practice sketching quadratic graphs.

On the axes below, complete the sketch for the quadratic graph.
You must label the roots (x intercepts) on your sketch.

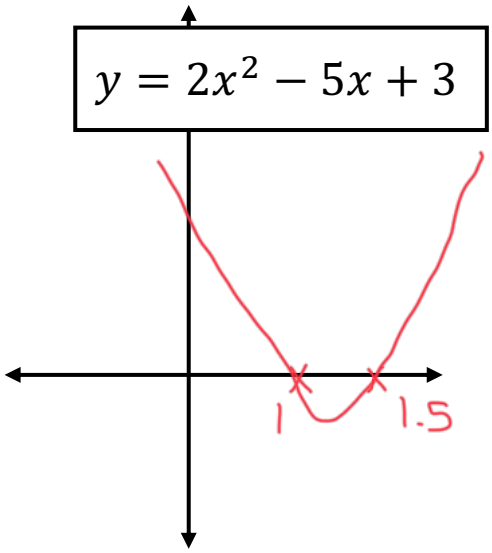
$y = 2x^2 + 7x + 3$





$y = 3x^2 + 5x + 2$

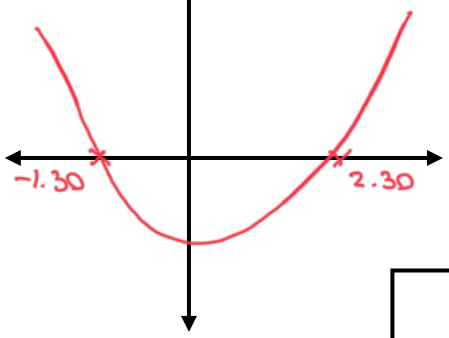
$y = 2x^2 - 5x + 3$

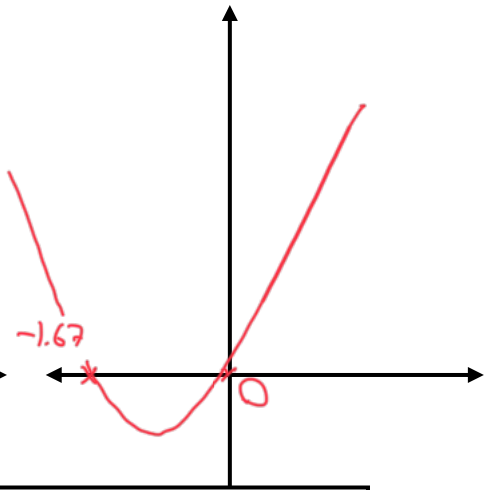


You'll need a calculator for these.
Write your roots to 2 decimal places.

Watch the Addvance video on solving quadratic equations.

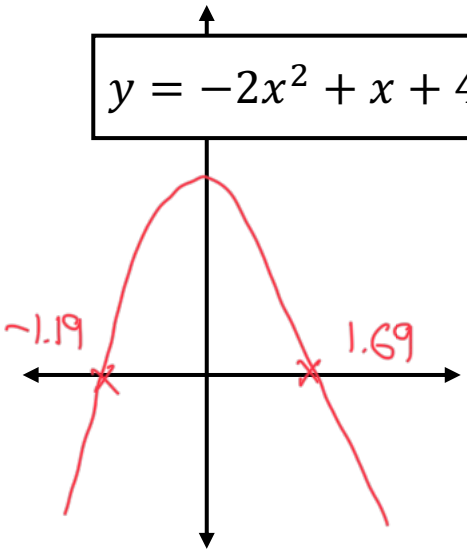
$y = x^2 - x - 3$





$y = 3x^2 + 5x$

$y = -2x^2 + x + 4$



Quadratic Inequalities

We're ready!

Let's solve some quadratic inequalities.

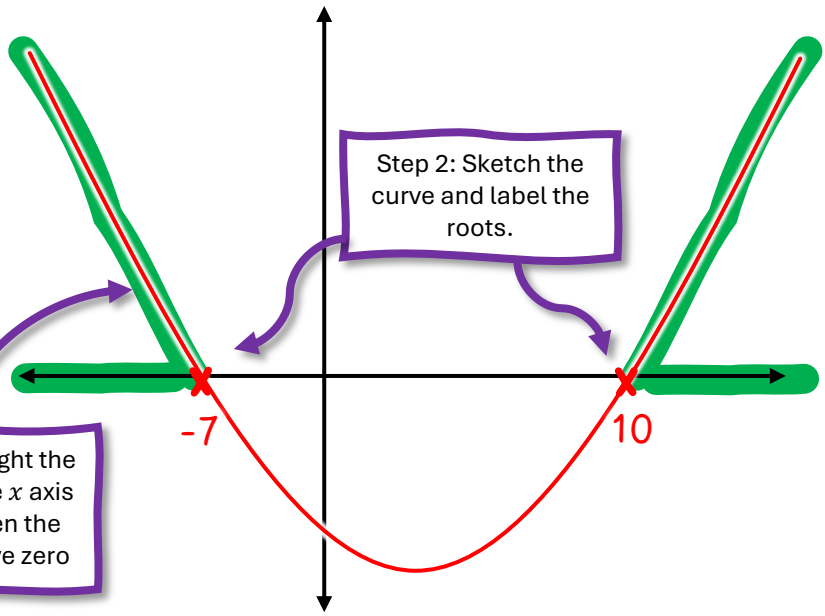
Example: Solve $x^2 - 3x - 70 > 0$

Step 1: Solve the quadratic for when it is equal to zero.

$x^2 - 3x - 70 = 0$

$(x + 7)(x - 10) = 0$

$x = -7 \quad x = 10$



Step 2: Sketch the curve and label the roots.

Step 3: Highlight the curve and the x axis to show when the curve is above zero

$x < -7 \quad x > 10$

Step 4: Write the x values as an inequality, or as two separate inequalities.

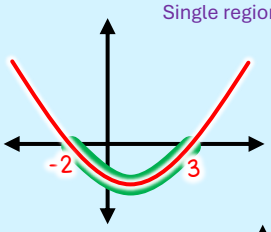
Tip 1!

There are two different forms of inequality for the final answer.

Single region, single inequality

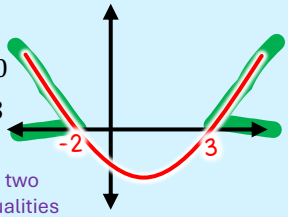
$x^2 + x - 6 < 0$

$-2 < x < 3$



$x^2 + x - 6 > 0$

$x < -2, x > 3$



Two regions, two separate inequalities

Tip 2!

✅ *Match the sign!*

If the question is "equal-to-ish" (≤ or ≥), your answer should be "equal-to-ish" too.

Your turn!

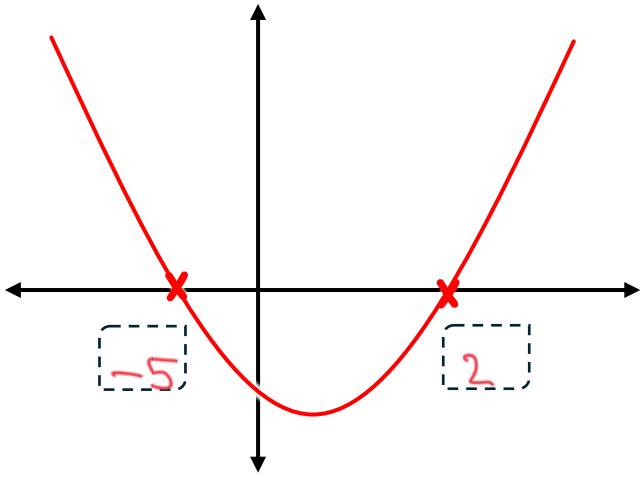
Fill in the blanks:

$x^2 + 3x - 10 < 0$

$x^2 + 3x - 10 = 0$

$(x \; +5)(x \; -2) = 0$

$x = -5 \quad x = 2$



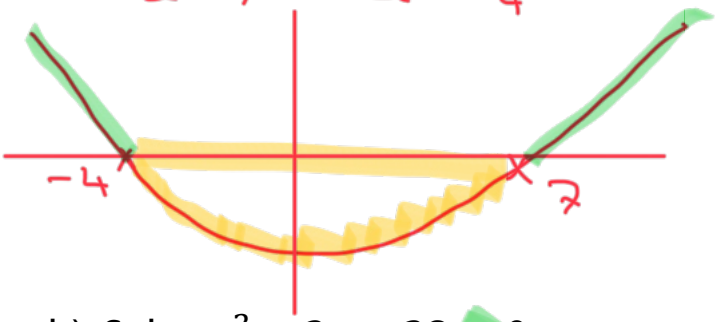
$-5 < x < 2$

Quadratic Inequalities

1. a) Solve $x^2 - 3x - 28 < 0$

$(x-7)(x+4)<0$

$x=7$ $x=-4$



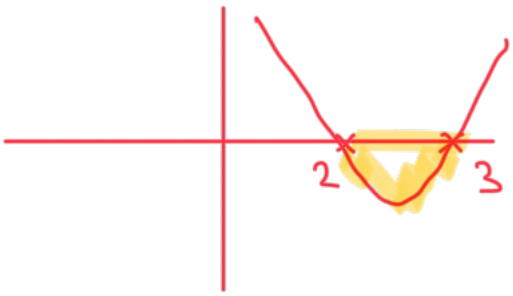
$-4 < x < 7$

b) Solve $x^2 - 3x - 28 > 0$

$x < -4$, $x > 7$

2. a) Solve $x^2 - 5x + 6 \leq 0$

$(x-3)(x-2) \leq 0$



$2 \leq x \leq 3$

b) Solve $x^2 - 5x + 6 \geq 0$

$x \leq 2$, $x \geq 3$

Quadratic Inequalities

3. a) Solve $x^2 - 5x < 0$

$0 < x < 5$

b) Solve $x^2 - 5x > 0$

$x < 0, x > 5$

4. a) Solve $x^2 - 81 \leq 0$

$-9 \leqslant x \leqslant 9$

b) Solve $x^2 - 81 \geq 0$

$x \leqslant -9, x \geqslant 9$

Quadratic Inequalities

These quadratics cannot be solved by factorising, you will need to use your calculator to solve them. Round all answers to 2 decimal places.

5. a) Solve $2x^2 - 5x + 1 < 0$

$0.22 < x < 2.28$

b) Solve $2x^2 - 5x + 1 > 0$

$x < 0.22$, $x > 2.28$

6. a) Solve $5x - 3x^2 \leq 0$

$x(5-3x) \leqslant 0$

$x = 0$, $x = \frac{5}{3}$

$0 \leqslant x \leqslant \frac{5}{3}$

b) Solve $5x - 3x^2 \geq 0$

$x \leqslant 0$, $x \geqslant \frac{5}{3}$

Quadratic Inequalities

7. a) Solve $3x^2 - 7x - 3 \leq 0$

$-0.37 \leq x \leq 2.70$

b) Solve $3x^2 - 7x - 3 \geq 0$

$x \leq -0.37, \quad x \geq 2.70$

8. a) Solve $4x^2 - 25 < 0$

$(2x - 5)(2x + 5) < 0$

$x = \frac{5}{2}, \quad x = -\frac{5}{2}$

$-2.5 < x < 2.5$

b) Solve $4x^2 - 25 > 0$

$x < -2.5, \quad x > 2.5$

Quadratic Inequalities

9. a) Solve $x^2 + 10 \leq 7x$

$x^2 - 7x + 10 \leq 0$

$(x-5)(x-2) \leq 0$

$2 \leqslant x \leqslant 5$

b) Solve $x^2 + 10 \leq 7x$

$x \leqslant 2, \; x \geqslant 5$

10. a) Solve $x^2 + 10x < 24$

$x^2 + 10x - 24 < 0$

$(x+12)(x-2) < 0$

$-12 < x < 2$

b) Solve $x^2 + 10x > 24$

$x < -12, \; x > 2$

6

Quadratic Inequalities

9. a) Solve $5x(x+1) \leq 14x+1$

$5x^2-9x-1 \leq 0$

$-0.10 \leq x \leq 1.90$

b) Solve $5x(x+1)14x+1 \geq 0$

$x \leq -0.1 \ , \ x \geq 1.9$

10. a) Solve $9x^2 < 144$, leaving your inequality in terms of fractions.

$9x^2-144<0$

$(3x-12)(3x+12)<0$

$x=4 \ , \ x=-4$

$-4<x<4$

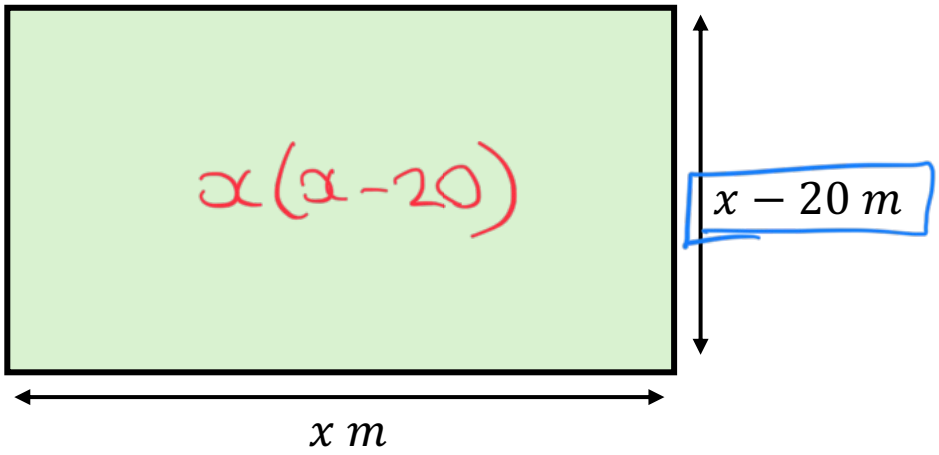
b) Solve $9x^2 \geq 144$

$x<-4, \ x>4$

Quadratic Inequalities

Applied problem solving.

13. A rectangular garden is drawn below.



The width is 20 metres shorter than the length.

The gardener wants the area to be **at most** $300\ m^2$.

For what values of x will the area be at most $300\ m^2$?

$x(x-20) \leqslant 300$

$x^2 - 20x \leqslant 300$

$x^2 - 20x - 300 \leqslant 0$

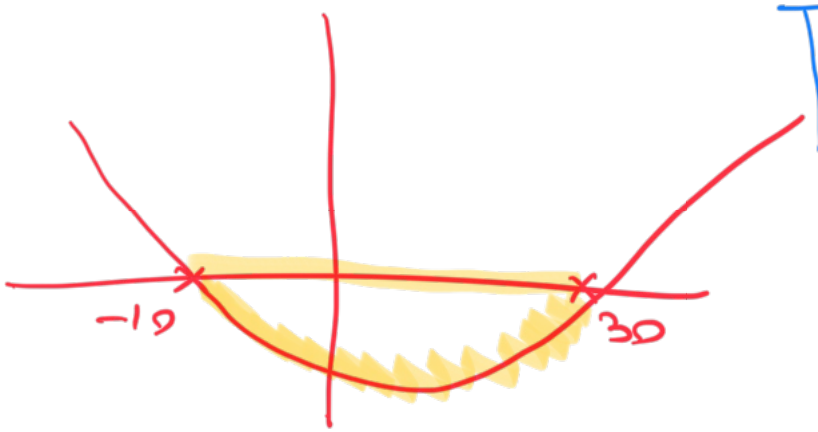
$(x-30)(x+10) \leqslant 0$

$x = 30, \quad x = -10$

$-10 \leqslant x \leqslant 30$

In context, $x > 20$

$20 < x \leqslant 30$



Quadratic Inequalities

14. Jim thinks of an integer. Jim adds 5 to the integer and then multiplies this by his original number.

His answer is less than 36.

List all the possible integers that Jim could have been thinking off.

$n(n+5) < 36$

$n^2 + 5n < 36$

$n^2 + 5n - 36 < 0$

$(n+9)(n-4) < 0$

$n = -9, \; n = 4$

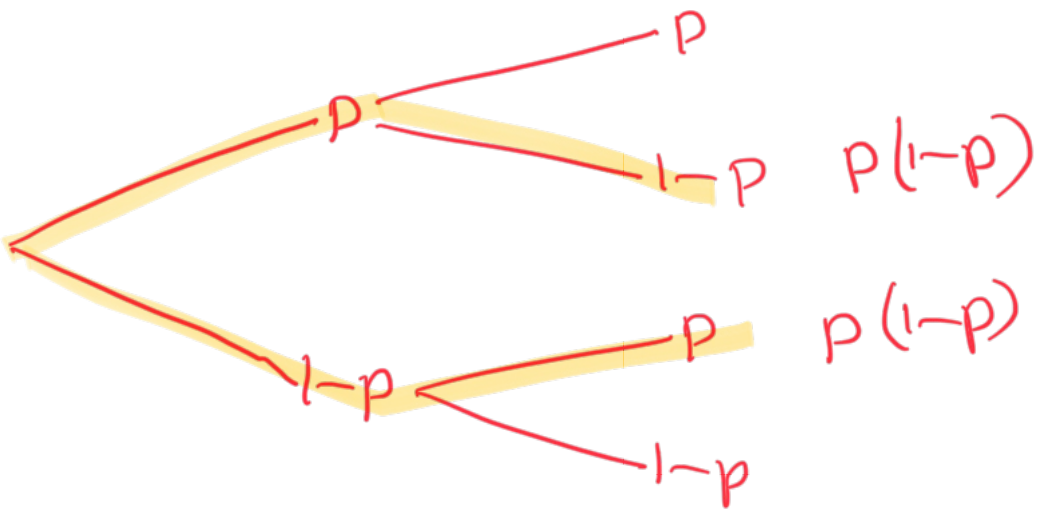
$-9 < n < 4$

$-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$

15. The probability of Fred scoring winning a game is always p. Fred plays the game twice in a row.

The probability of him winning one game but loosing the other is 0.2 or less.

What are the possible values of p?



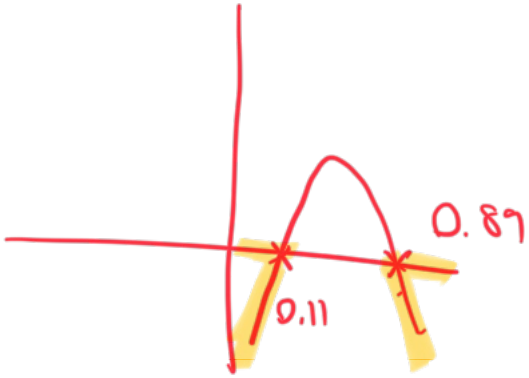
$2p(1-p) \leqslant 0.2$

$2p - 2p^2 \leqslant 0.2$

$p - p^2 \leqslant 0.1$

$-p^2 + p - 0.1 \leqslant 0$

$p = 0.89, \quad 0.11$



$0 \leqslant p \leqslant 0.11$

or

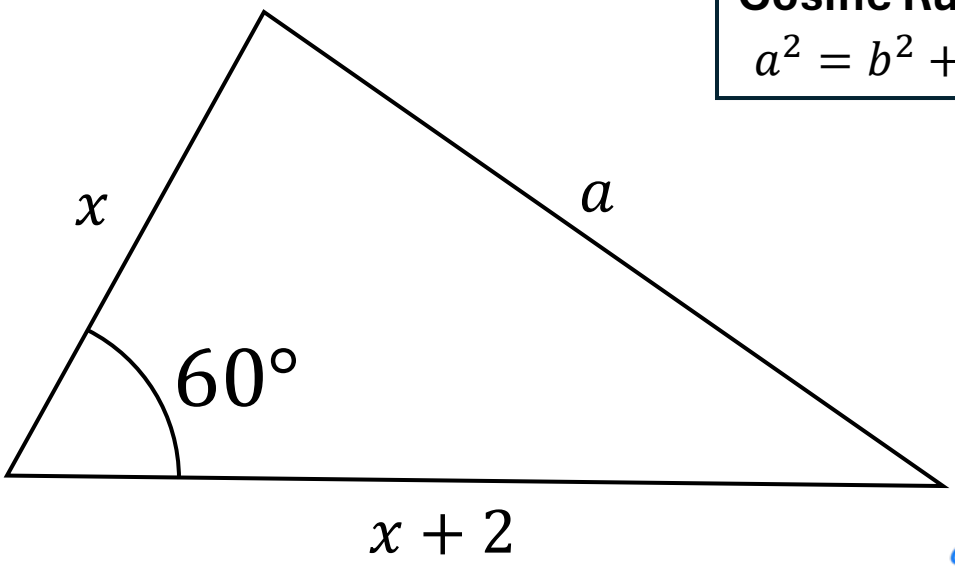
$0.89 \leqslant p \leqslant 1$

16. Look at the triangle below.

Find the values of x such that a is less than 10.

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$



$\cos(60) = \frac{1}{2}$

$10^2 \geq x^2 + (x+2)^2 - 2x(x+2)\cos(60)$

$100 \geq x^2 + x^2 + 4x + 4 - x^2 - 2x$

$0 \geq x^2 + 2x - 96$

$x^2 + 2x - 96 \leq 0$

$x = 8.84 \quad x = -10.84$

$-10.84 \leq x \leq 8.84$

In context:

$0 < x \leq 8.84$