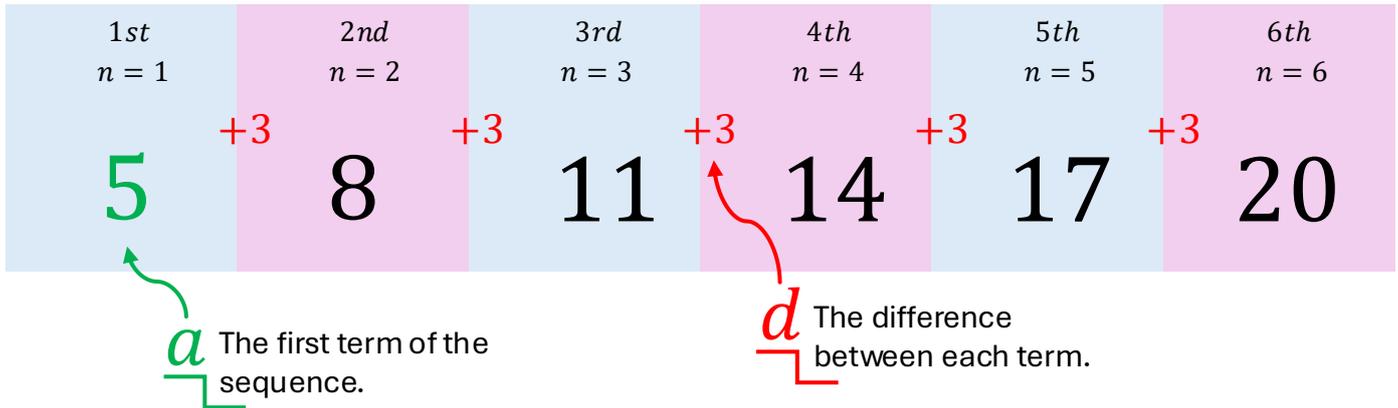




An **arithmetic sequence** is a list of numbers in which **each term after the first is obtained by adding a constant value**, called the **common difference**.



An **arithmetic series** is the **sum of the terms of an arithmetic sequence**.

You should know the two equations below.

$$U_n = a + (n - 1)d$$

U_n The n^{th} term.

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

S_n The sum of the first n terms of the arithmetic series.

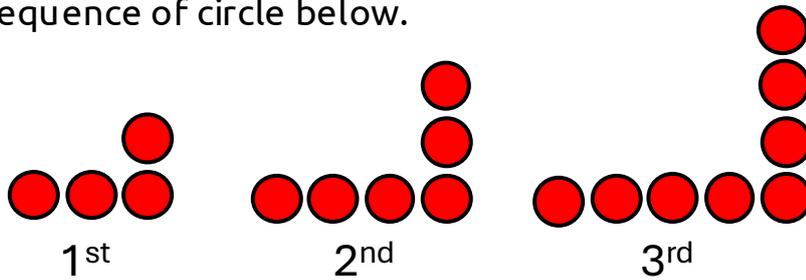
1 Complete the table below for each arithmetic sequence. The first row has been done for you.

a	d	Next 4 Terms				10 th Term	Sum of the first 10 terms.
		$n = 2$	$n = 3$	$n = 4$	$n = 5$		
70	4	74	78	82	86	$70 + (10 - 1) \times 4 = 106$	$\frac{10}{2} [2 \times 70 + (10 - 1) \times 4]$ $= 880$
6	5	11	16	21	26	51	285
200	-8	192	184	176	168	128	1640
11	3	14	17	20	23	38	245
50	-3	47	44	41	38	23	365

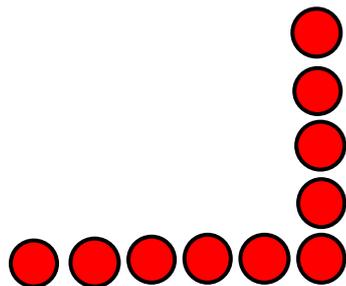


Grade 5 questions

2 Look at the sequence of circle below.



a) Draw a diagram of the 4th term of the sequence.



.....(1)

b) How many circles will be in the 5th shape of the sequence?

12
.....(1)

c) Write an expression in terms of n for the number of circles in the n^{th} term of the sequence?

$2n+2$
.....(1)

d) How many circle will be in the 30th term of the sequence?

62
.....(1)

e) How many circles will there be in total in the first 8 terms of the sequence?

$4(2(4)+7(2))=88$

88
.....(2)



- 3** The second term of an arithmetic sequence is 7 and the third term is 10.

Work out the sum of the first 20 terms of the series.

$$a+d=7 \quad a+2d=10$$

$$d = 3, a = 4$$

$$\dots\dots\dots 650 \dots\dots\dots(3)$$

- 4** The first term of an arithmetic sequence is 10, and the 15th term is 80.

Work out the sum of the first 15 terms.

$$a = 10 \quad a + 14d = 80$$

$$14d = 70$$

$$d = 5$$

$$\dots\dots\dots 675 \dots\dots\dots(3)$$

- 5** An arithmetic sequence is defined by:

$$U_n = 4n - 3$$

- a) Which term in the sequence has a value of 117?

$$\dots\dots\dots n = 30 \dots\dots\dots(1)$$

- b) What is the sum of the first 25 terms?

$$\dots\dots\dots 1225 \dots\dots\dots(2)$$



Grade 7 questions

6 Mario saves money in the following pattern:

January: £50

February: £55

March: £60

He increases his savings by £5 each month.

After m months he saves a total of **£3290**.

a) Show that $6580 = m(5m + 95)$.

$$a = 50, d = 5 \quad 3290 = \frac{m}{2}(2(50) + (m-1)5)$$

$$6580 = m(100 + 5m - 5)$$

$$6580 = m(5m + 95) \quad \text{As required} \dots\dots\dots(2)$$

b) Hence, calculate the value of m .

$$6580 = 5m^2 + 95m$$

$$0 = 5m^2 + 95m - 6580$$

$$0 = 5m^2 + 95m - 6580$$

$$5(m+47)(m-28) = 0$$

$$m = -47 \text{ \& } m = 28$$

$$m = 28 \text{ (m can't be negative)}$$

$$m = 28 \dots\dots\dots(3)$$

7 In an arithmetic series:

$$U_6 = 40 \quad \& \quad U_{11} = 75$$

Work out the sum of the first 20 terms. $U_n = a + (n-1)d$

$$40 = a + (6-1)d$$

$$40 = a + 5d$$

$$75 = a + 10d$$

$$- (40 = a + 5d)$$

$$75 = a + (11-1)d$$

$$75 = a + 10d$$

$$35 = 5d \quad d = 7 \quad \therefore a = 5$$

$$S_{20} = \frac{20}{2} (2 \times 5 + (20-1)(7)) = 10(143) = 1430$$

$$a = 5 \dots\dots\dots(4)$$



8 In an arithmetic series:

$$S_8 = 108 \quad \& \quad S_{16} = 376$$

a) Find the value of the common difference.

$$108 = \frac{8}{2} (2a + (8-1)d)$$

$$108 = 4(2a + 7d)$$

$$108 = 8a + 28d$$

$$216 = 16a + 56d$$

$$376 = \frac{16}{2} (2a + (16-1)d)$$

$$376 = 8(2a + 15d)$$

$$376 = 16a + 120d$$

$$16a + 120d = 376$$

$$-(16a + 56d = 216)$$

$$64d = 160$$

$$d = 2.5$$

$$d = 2.5 \dots\dots\dots(3)$$

b) Find the value of the first term.

$$108 = 8a + 28(2.5)$$

$$108 = 8a + 70$$

$$8a = 38$$

$$a = 4.75$$

$$a = 4.75 \dots\dots\dots(2)$$

9 In an arithmetic series:

$$U_2 = 8.5 \quad , \quad U_5 = 13 \quad \& \quad S_N = 292$$

Find the value of N .

$$8.5 = a + (2-1)d$$

$$8.5 = a + d$$

$$13 = a + (5-1)d$$

$$13 = a + 4d$$

$$13 = a + 4d$$

$$-(8.5 = a + d)$$

$$3d = 4.5$$

$$d = 1.5$$

$$8.5 = a + 1.5$$

$$a = 7$$

$$292 = \frac{n}{2} (2 \times 7 + (n-1)(1.5))$$

$$292 = \frac{n}{2} (14 + 1.5n - 1.5)$$

$$584 = n(14 + 1.5n - 1.5)$$

$$584 = 14n + 1.5n^2 - 1.5n$$

$$1168 = 28n + 3n^2 - 3n$$

$$1168 = 3n^2 + 25n$$

$$3n^2 + 25n - 1168 = 0$$

$$(3n + 73)(n - 16) = 0$$

$$n = \frac{-73}{3}$$

$$n = 16$$

$$n = 16 \dots\dots\dots(5)$$



10 An arithmetic sequence is:

2, 9, 16, 23, 30,...

Work out the sum of the terms from the 41st to the 100th term inclusive*.

**This means you add the 41st, 42nd, 43rd, ..., terms all the way up to the 100th term.*

$$S_n = \frac{n}{2} (2a + (n-1)d) \quad a: 2 \quad d: 9-2 = 7 \quad n: 100 \text{ \& \ } 40$$

$$S_{40} = \frac{40}{2} (2 \times 2 + (40-1)(7)) = 20(277) = 5540$$

$$S_{100} = \frac{100}{2} (2 \times 2 + (100-1)(7)) = 50(697) = 34850$$

$$S_{41-100} = 34850 - 5540 = 29310$$

29310
.....(5)

Grade 9 questions

11 The interior angles of a polygon with n sides form an arithmetic sequence.

Largest angle = 172°

Common difference = -4°

Find the number of sides n (given that $n > 5$).

$$S_n = \frac{n}{2} (2a + (n-1)d) \quad a: 172 \quad d: -4 \quad S_n = 180(n-2)$$

$$180(n-2) = \frac{n}{2} (2 \times 172 + (n-1)(-4))$$

$$360(n-2) = n (344 + 4 - 4n)$$

$$360n - 720 = 348n - 4n^2$$

$$4n^2 - 348n + 360n - 720 = 0$$

12, Dodecagon

$$4n^2 + 12n - 720 = 0 \quad n^2 + 3n - 180 = 0$$

.....(5)

$$(n - 12)(n - 15) = 0$$

$$n = -15$$

$$n = 12$$



12 The first three terms of an arithmetic series are:

$$k, \quad 2k + 3, \quad 4k - 2$$

(a) Show that $k = 8$.

$$\begin{aligned} 2k + 3 - k &= 4k - 2 - (2k + 3) & k + 3 &= 2k - 5 \\ 3 + 5 &= 2k - k & k &= 8 \end{aligned}$$

.....(3)

(b) Find the sum of the first 30 terms of the series.

$$\begin{aligned} S_n &= \frac{n}{2} (2a + (n-1)d) \quad a: 8 \quad d: 2(8) + 3 - 8 = 11 \quad n: 30 \\ S_n &= \frac{30}{2} (2 \times 8 + (30-1)(11)) = 15(16 + 319) \\ S_n &= 5025 \end{aligned} \quad \text{.....(2)}$$

13 Here are the first three terms of an arithmetic sequence.

$$8j, \quad 7j - 3, \quad 4j + 2$$

The sum of the first n terms of the sequence is -1914 .

Find the value of n .

$$\begin{aligned} 7j - 3 - 8j &= 4j + 2 - (7j - 3) & -j - 3 &= -3j + 5 \\ -j + 3j &= 5 + 3 & 2j &= 8 & j &= 4 \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2} (2a + (n-1)d) \\ a: 8(4) &= 32 \quad d: 7(4) - 3 - 8(4) = -7 \quad S_n = -1914 \\ -1914 &= \frac{n}{2} (2 \times 32 + (n-1)(-7)) \quad -3828 = n(64 + 7 - 7n) \\ -3828 &= 71n - 7n^2 \quad 7n^2 - 71n - 3828 = 0 \\ (7n + 132)(n - 29) &= 0 & \text{.....(5)} \\ n &= \frac{-132}{7} \quad n = 29 \end{aligned}$$



Challenge Question

- 14** The sum of the first **40** terms of an arithmetic series is **4 times** the sum of the first **30** terms of the same series.

Find the sum of the first **25** terms of this series.

$$S_n = \frac{n}{2} (2a + (n-1)d) \quad a: 2 \quad d: 9-2 = 7 \quad n: 100 \text{ \& \ } 40$$

$$S_{40} = 4(S_{30})$$

$$\frac{40}{2} (2a + (40-1)(d)) = 4 \left[\frac{30}{2} (2a + (30-1)(d)) \right]$$

$$20 (2a + 39d) = 4 [15(2a + 29d)]$$

$$40a + 780d = 60(2a + 29d)$$

$$40a + 780d = 120a + 1740d$$

$$40a - 120a = 1740d - 780d$$

$$-80a = 960d$$

$$a = -12d$$

$$S_{25} = \frac{25}{2} (2(-12d) + (25-1)(d))$$

$$S_{25} = \frac{25}{2} (-24d + 24d)$$

$$S_{25} = \frac{25}{2} (0)$$

$$S_{25} = 0$$



Full Mark Scheme
and Revision
Videos

0.....(5)